제 31회 한국반도체학술대회
The 31st Korean Conference on Semiconductors

Implementation of Zero-noise Extrapolation in 28Si/SiGe Spin Qubits

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Presenter: Jaewon Jung

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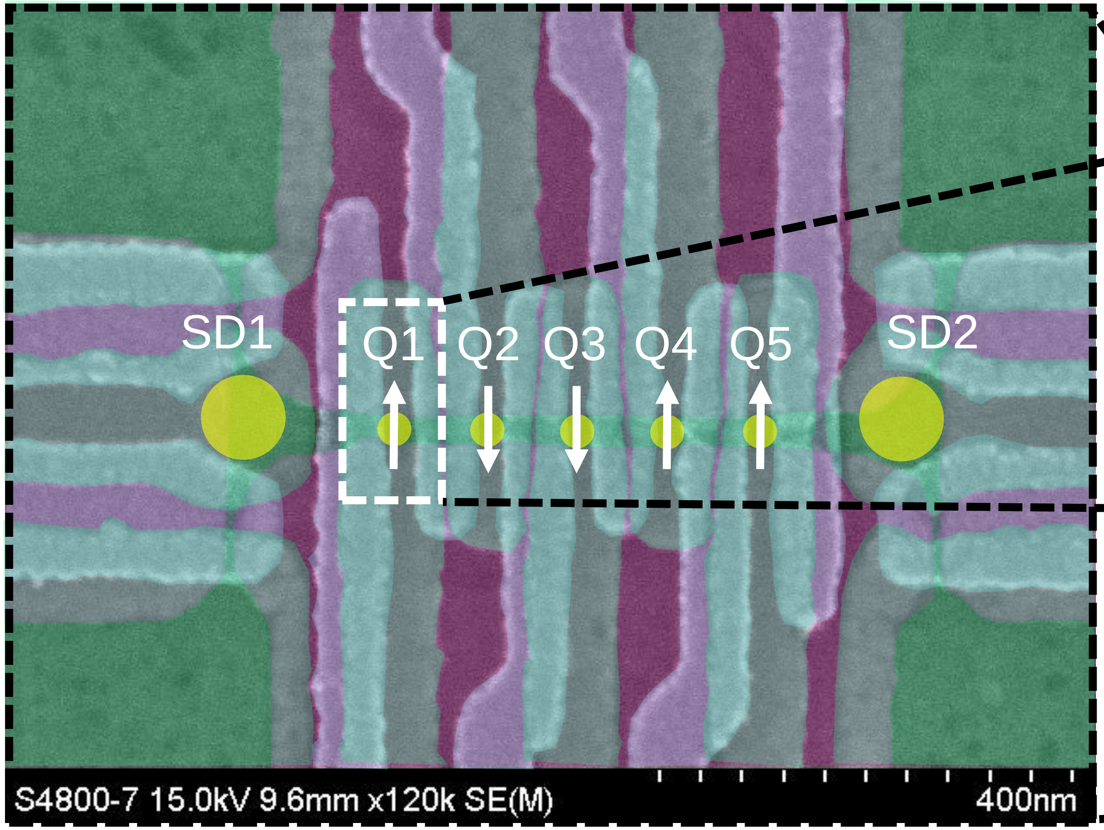
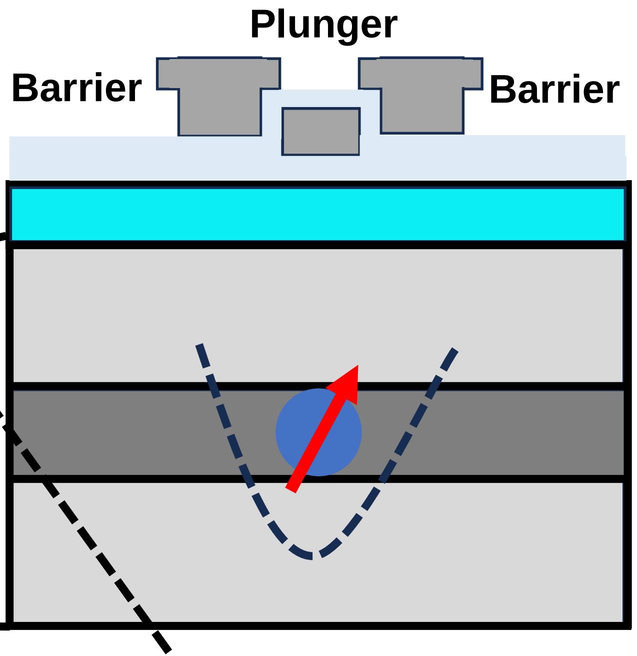
1.1 Intro to 28Si/SiGe Spin Qubits

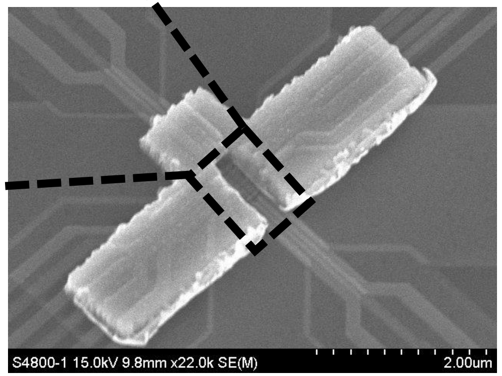


False-colored SEM image of the device

Courtesy of J. Park



Si or SiOx cap	**~10nm**
$Si_{0.7}Ge_{0.3}$	**30~40nm**
28Si well (2DEG)	**8~10nm**
$Si_{0.7}Ge_{0.3}$	**1~4μm**

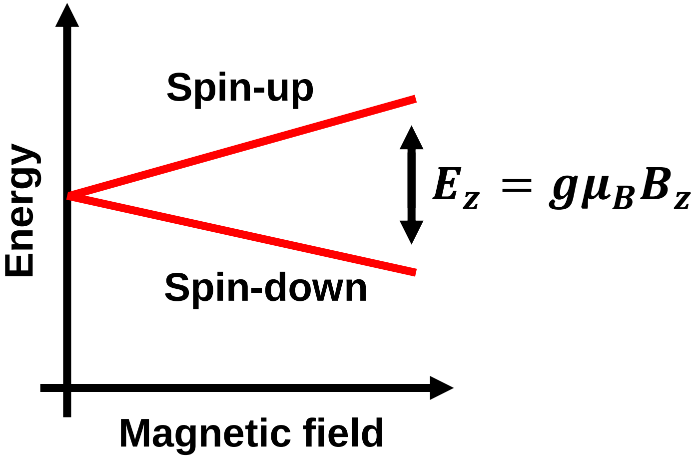


Cobalt micromagnet for manipulating spins

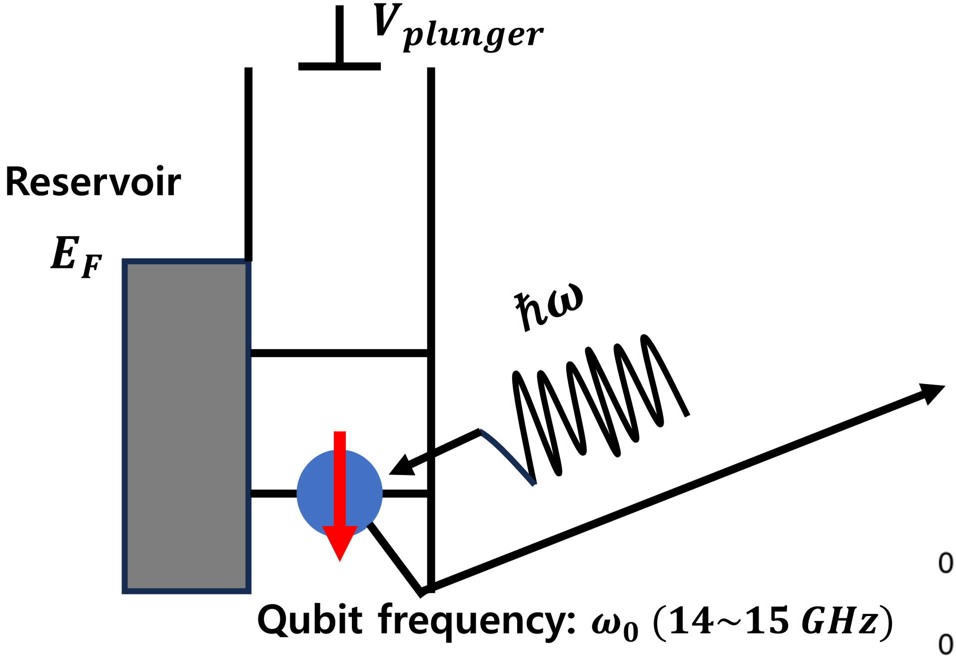
Due to stray E-field
Unstable charge traps
-> 1/f Charge noise

1.2 Gate operation and measurement of single spin qubit

Zeeman splitting



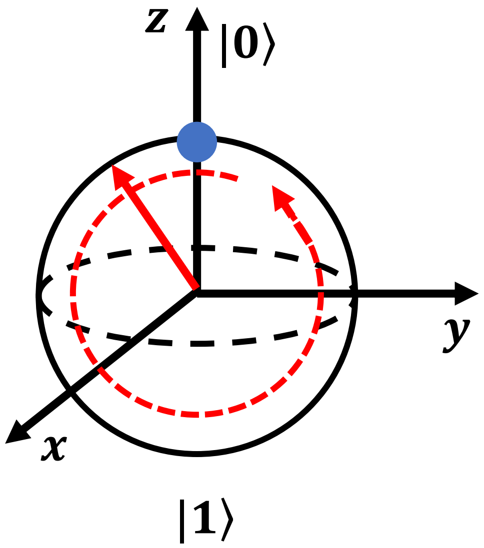
Resonant single-spin control

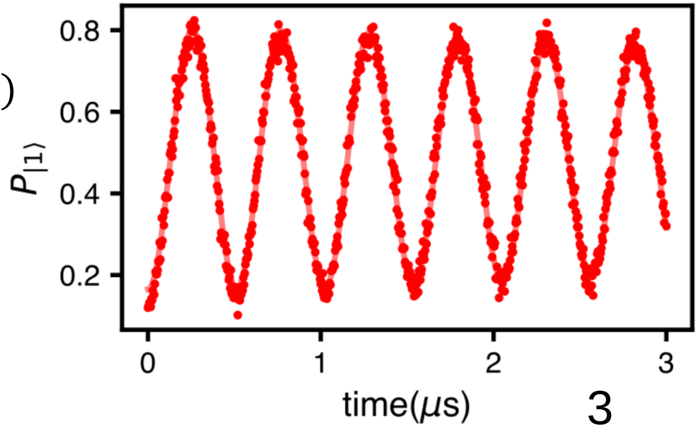


$$\hat{H}_{rot} = \frac{\hbar}{2}(\omega_0 - \omega)\ \hat{\sigma}_z + \frac{\hbar\gamma}{2}\ \hat{\sigma}_x$$

Effective Hamiltonian in rotating frame

Rabi oscillation

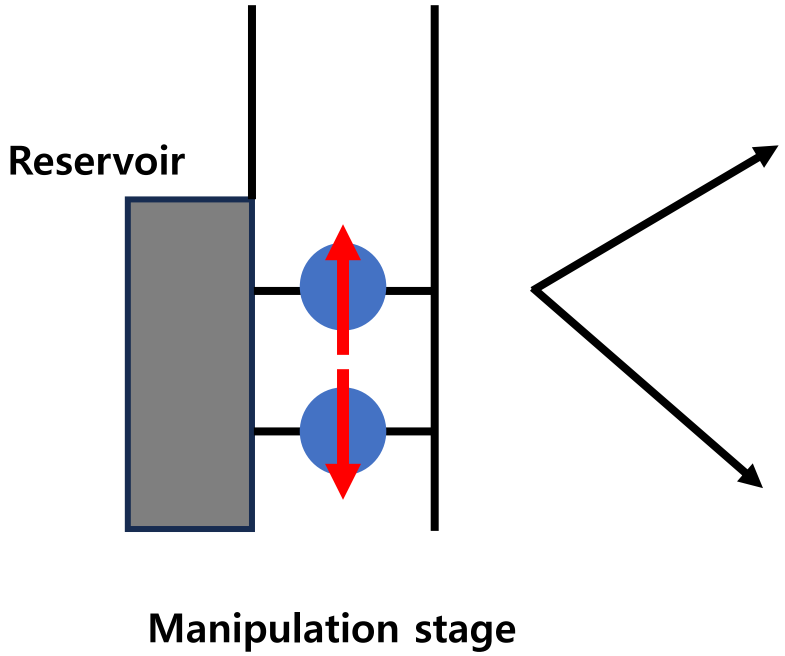


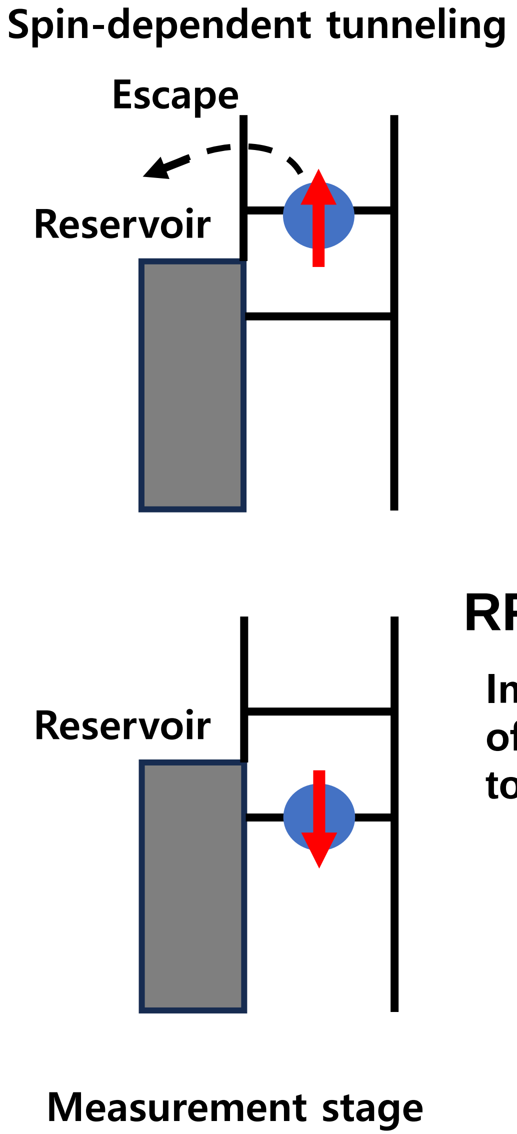


Loss, Daniel, and David P. DiVincenzo.,
Physical Review A 57, no. 1, 120-26 (1998)

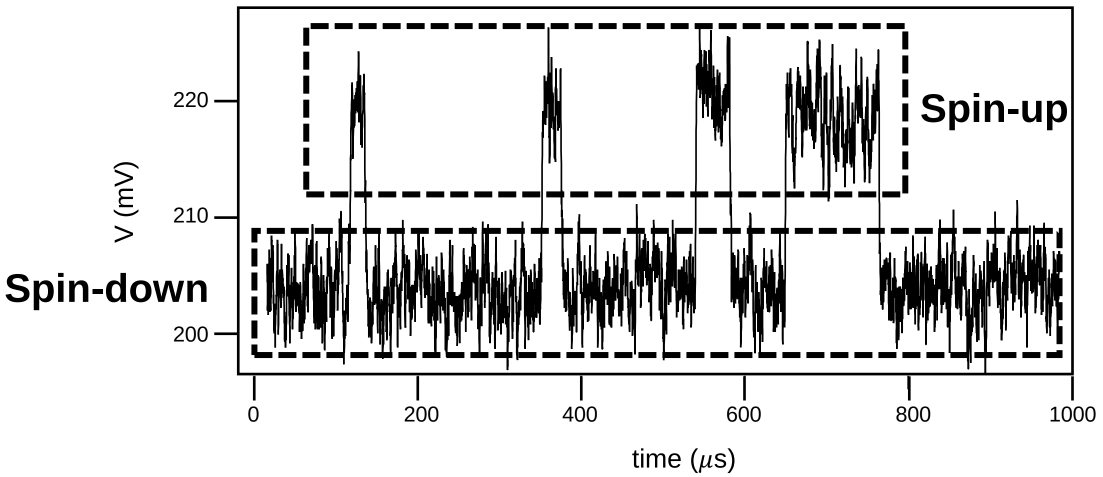
1.2 Gate operation and measurement of single spin qubit

Energy selective tunneling single-shot measurement (EST)



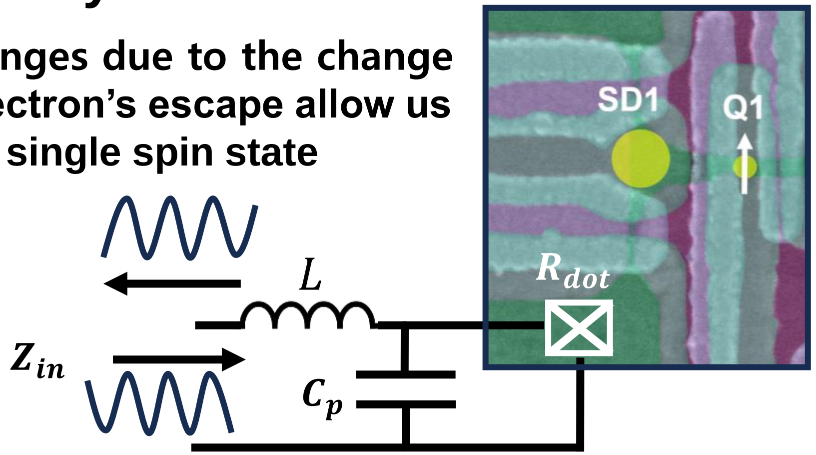


Courtesy of H. Sohn



RF reflectometry

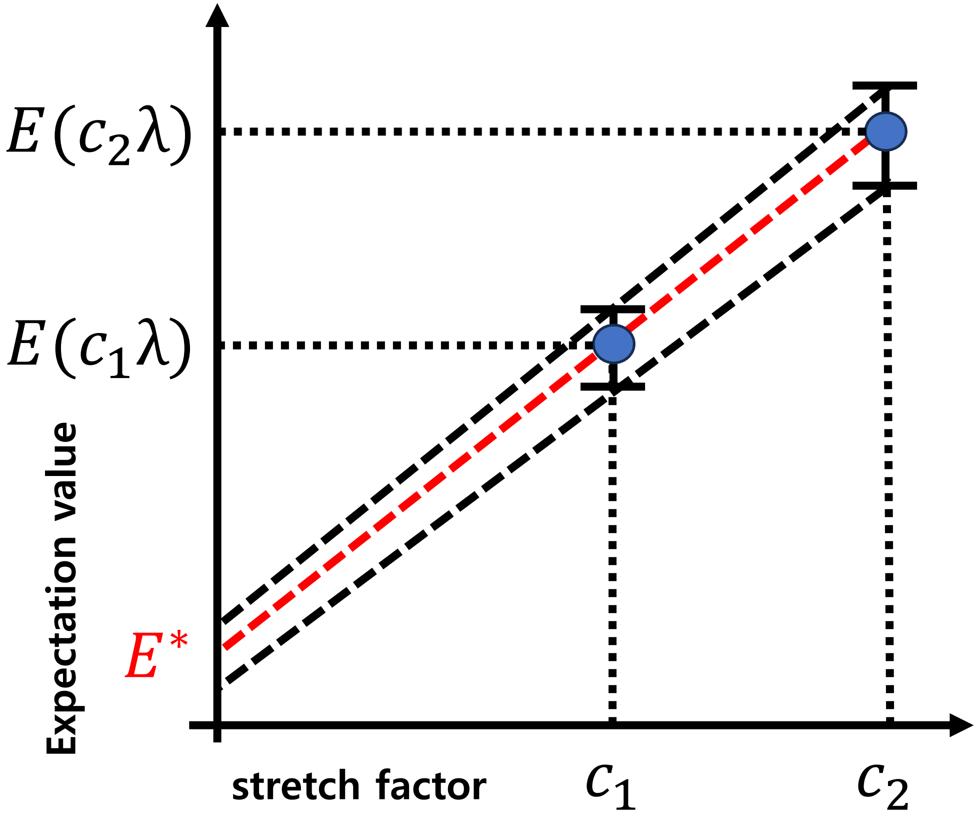
Impedance changes due to the change of R_{dot} from electron's escape allow us to measure the single spin state



Jehyun Kim, Jonginn Yun, Wonjin Jang, Hyeongyu Jang, Jaemin Park, Younguk Song, Min-Kyun Cho, Sangwoo Sim, Hanseo Sohn, Hwanchul Jung, Vladimir Umansky, and Dohun Kim
Phys. Rev. Lett. **129**, 040501 – Published 19 July 2022

2.1 Zero-noise extrapolation: Principle

Measure the noise-amplified results and extrapolate them to zero-noise limit



An example of a first-order Richardson extrapolation
(E^*: zero-noise value, $E(c_1\lambda), E(c_2\lambda)$: noise-amplified value)

$$H(t) = \sum_\alpha J_\alpha(t) P_\alpha$$

$J_\alpha(t)$: strength of interaction

P_α: N-qubit Pauli operator

Time dependent drive Hamiltonian

$$E_H(\lambda) = E^* + \sum_{k=1}^{n} a_k \lambda^k + O(\lambda^{n+1})$$

$E_H(\lambda)$: expectation value for a state evolved by *H(t)*
(λ: small noise parameter)

↓ **An improved approximation by Richardson extrapolation method**

$$\hat{E}_H^n(\lambda) = \sum_{i=0}^{n} \gamma_i \hat{E}_H(c_i\lambda)$$

n^{th}-order Richardson extrapolation estimate

For a chosen set of c_i and the coefficients γ_i

$$\sum_{i=0}^{n} \gamma_i = 0\,, \quad \sum_{i=0}^{n} \gamma_i c_i^k = 0$$

2.1 Zero-noise extrapolation: Optimization

By optimizing the noise stretch factor and the number of shots per experiment, we can obtain more accurate and stable mitigated result

$$\hat{E}_H^n(\lambda) = \sum_{i=0}^{n} \gamma_i \hat{E}_H(c_i\lambda)$$

n^{th}-order Richardson extrapolation estimate

Michael Krebsbach, Björn Trauzettel, and Alessio Calzona
Phys. Rev. A **106**, 062436 (2022)

$$Bias\left[\hat{E}_H^n(\lambda)\right] = (-1)^n E_{\lambda_0}^{(n+1)}(\xi)\frac{C_n}{(n+1)!}$$

$$Bias\left[\hat{E}_H^n(\lambda)\right] = E_H(\lambda) - E^*, \; C_n = \prod_{j=0}^{n} c_j$$

By using <u>adequate size of noise stretch factor</u>, we can reduce the <u>bias of mitigated result</u>

$$Var\left(\hat{E}_H^n(\lambda)\right) = \sum_{j=0}^{n} \gamma_j^2 \frac{\sigma^2}{N_j}$$

$$\gamma_j = \prod_{k\neq j} \frac{c_k}{c_k - c_j}$$

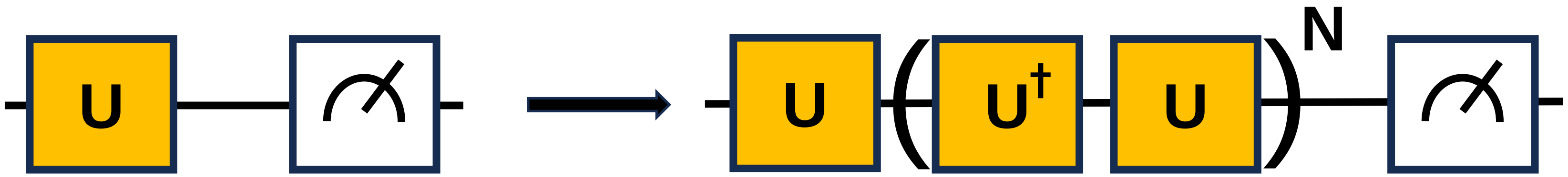
By using <u>adequate number of shots</u> per noise parameter, we can reduce the <u>variance of mitigated result</u>

2.1 Zero-noise extrapolation: Noise amplification

1. Digital noise scaling: Unitary folding

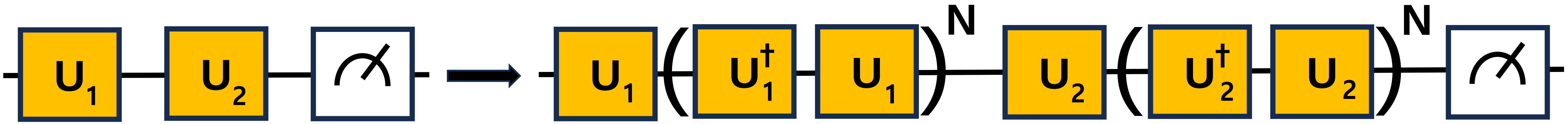
T. Giurgica-Tiron, Y. Hindy, R. LaRose, A. Mari, and W. J. Zeng, in 2020 IEEE International Conference on Quantum Computing and Engineering (QCE) (IEEE, Denver, CO, 2020), p. 306.

1. Global folding: Folding the whole circuit for N times



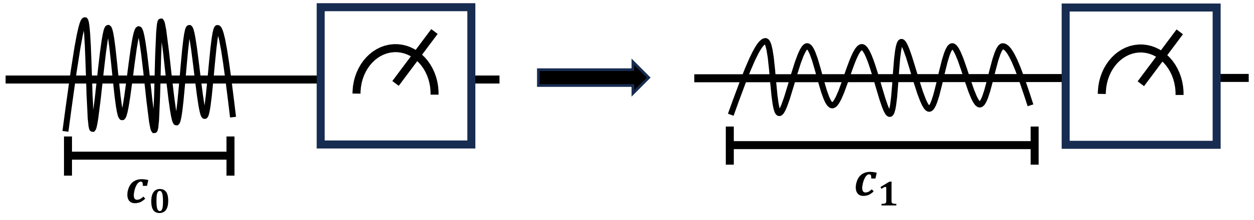
U: Unitary gate representing the whole quantum circuit

2. Local folding: Folding the each gate for N times



U: Unitary gate representing each single gate

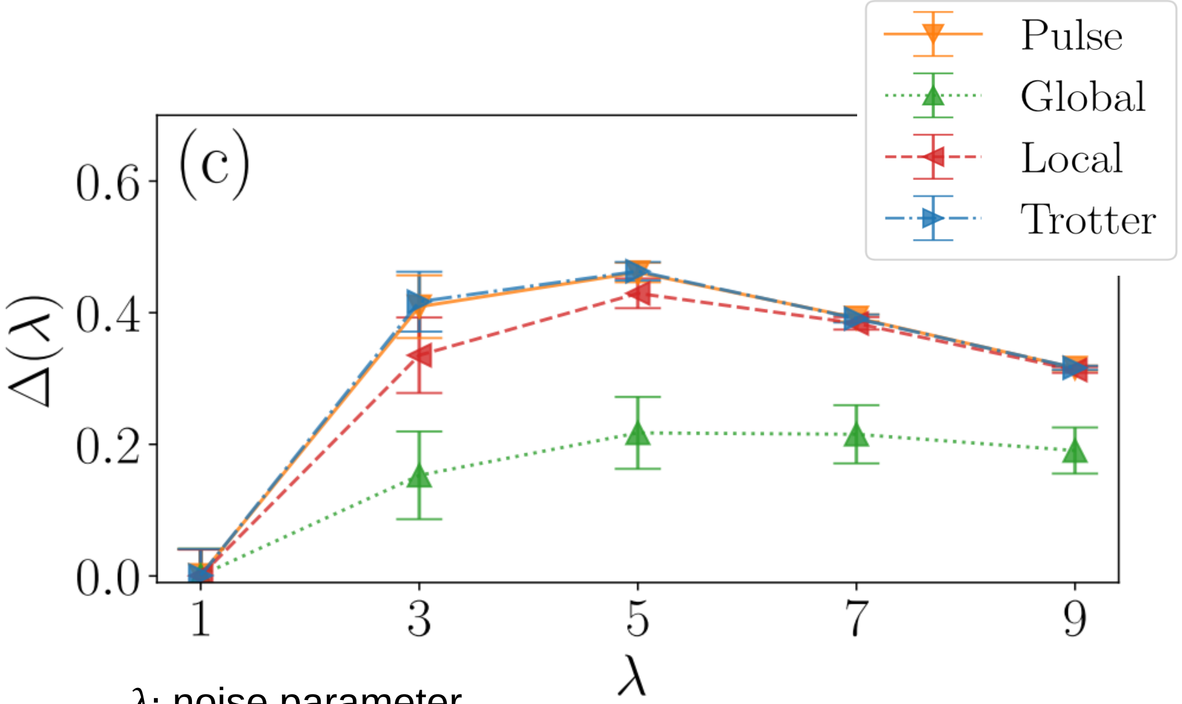
2. Analog noise scaling: Pulse stretching



Stretching the pulse for a desired stretch factor from c_0 to c_1

2.1 Zero-noise extrapolation: Noise spectrum

In ideal, noise should be invariant under <u>time-rescaling.</u>

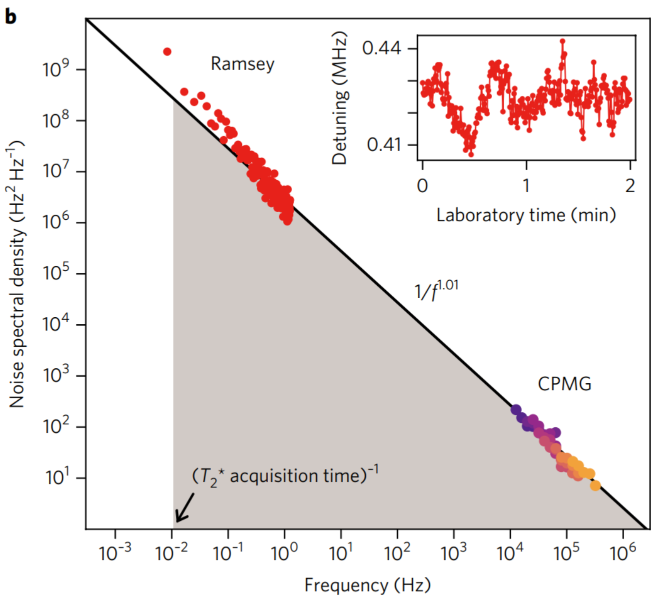


λ: noise parameter
Δ(λ): relative noise-scaling error

Kevin Schultz, Ryan LaRose, Andrea Mari, Gregory Quiroz, Nathan Shammah, B. David Clader, and William J. Zeng
Phys. Rev. A **106**, 052406 (2022)

For <u>white noise</u>, pulse-stretching method can be used to scale the noise ideally.

For <u>colored noise</u> $(1/f,\ 1/f^2)$, global folding method performs the best from the simulation result.



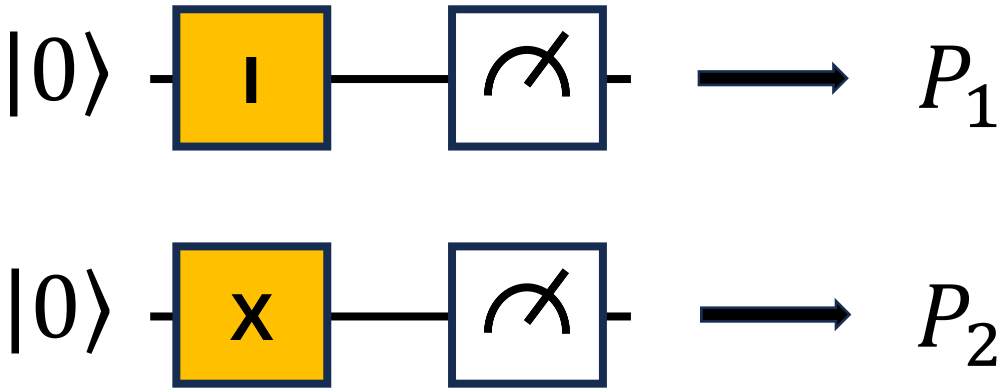
For silicon spin qubit, $1/f$ noise is a dominant noise source

Yoneda, J., Takeda, K., Otsuka, T. *et al.* A quantum-dot spin qubit with coherence limited by charge noise and fidelity higher than 99.9%. *Nature Nanotech* **13**, 102–106 (2018).

2.2 Readout error mitigation

Passive readout error mitigation: Mitigating local readout error for single qubit

Initialize



$$P_1 = F_1(1-\alpha) + (1-F_0)\alpha$$

$$\frac{P_2}{P_\pi} = F_1\alpha + (1-F_0)(1-\alpha)$$

$P_1(P_2)$: spin-up probability when prepared 0 (1) state
P_π: expected probability of spin-up considering the decoherence

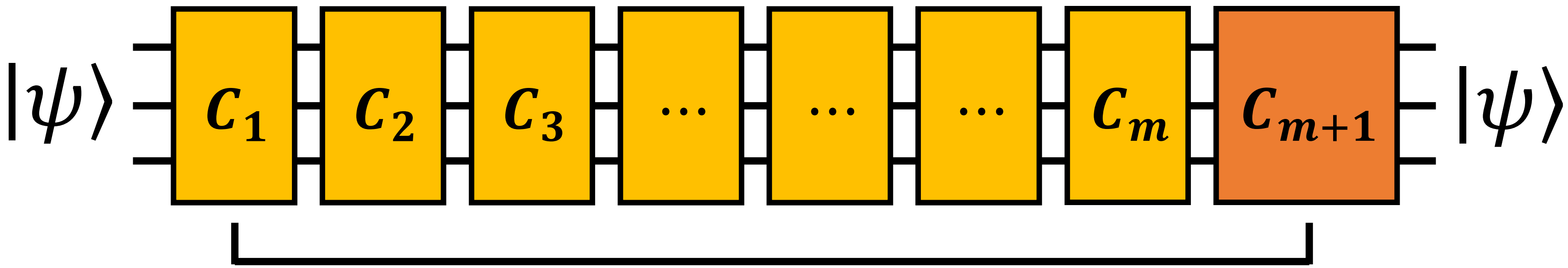
$$\longrightarrow \hat{C} = \begin{pmatrix} F_0 & 1-F_1 \\ 1-F_0 & F_1 \end{pmatrix} \longrightarrow P^* = \hat{C}^{-1}P^M$$

$\hat{C}$: response matrix
$F_0(F_1)$: fidelity of spin-down (up)

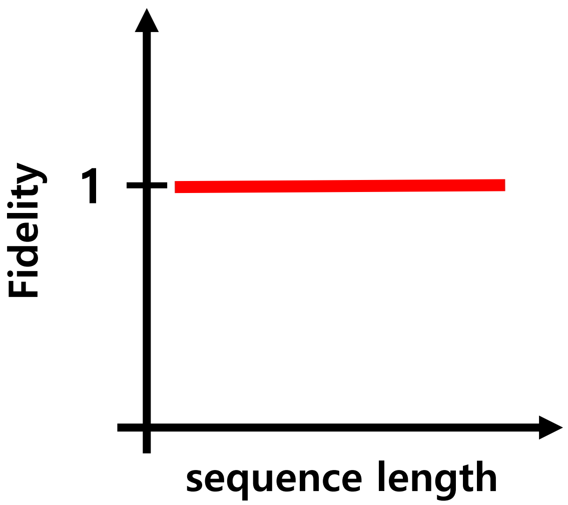
P^*: mitigated probabilities
P^M: measured probabilities

2.3 Randomized benchmarking

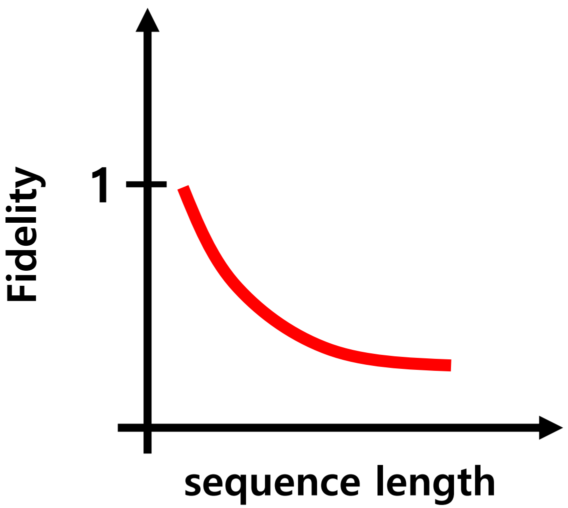
The benchmarking protocol which estimates average gate fidelity



= Identity, m: sequence length



Fidelity vs. sequence length
(Ideal case without gate error)



Fidelity vs. sequence length
(Non-ideal case with gate error)

$$F(m) = Ap^m + B$$

$$F_{ave} = p + \frac{1-p}{d} \quad (d = 2^n \text{ for n-qubit})$$

Extract average gate fidelity from exponential decay curve!

E. Magesan, J. M. Gambetta, and J. Emerson, Characterizing quantum gates via randomized benchmarking, Phys. Rev. A 85, 042311 (2012).

2.4 Quantum state tomography

The method to measure the fidelity of a quantum state

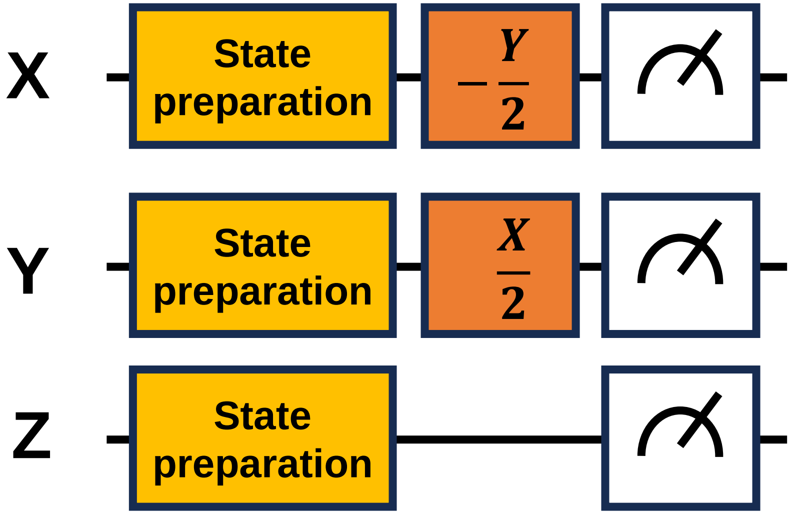
$$\hat{\rho} = \frac{1}{2}\sum_{i=0}^{3} S_i \hat{\sigma}_i$$

$\hat{\rho}$: density matrix of a quantum state
$\hat{\sigma}_i$: pauli matrix
Stokes parameter: $\{S_0, S_1, S_2, S_3\}$
$S_i \equiv Tr(\hat{\sigma}_i \hat{\rho}) = 2P_{|\psi\rangle} - 1$ for single qubit

$$F = \left\langle \psi_{theory} \middle| \rho_{exp} \middle| \psi_{theory} \right\rangle$$

F: fidelity of a quantum state

Experiment for measuring stokes parameter

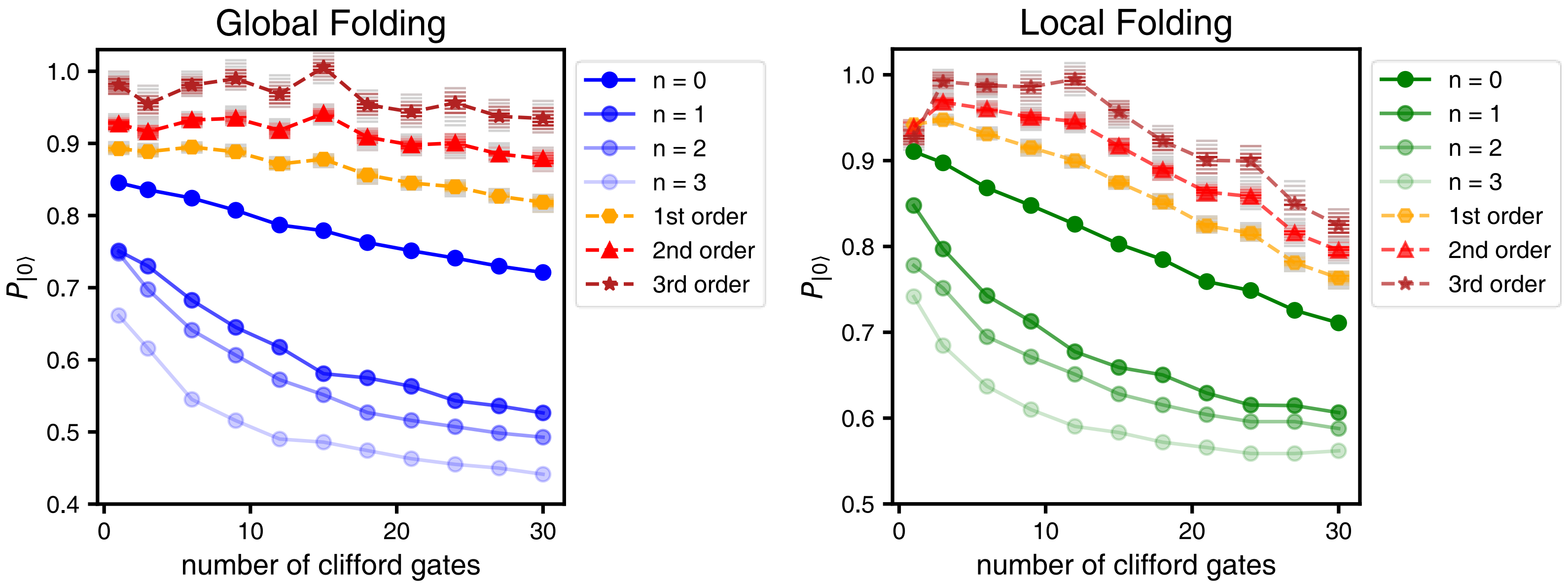


Altepeter, J., Jefrey, E. & Kwiat, P. Photonic state tomography. Adv. At. Mol. Opt. Phys. 52, 105–159 (2005).

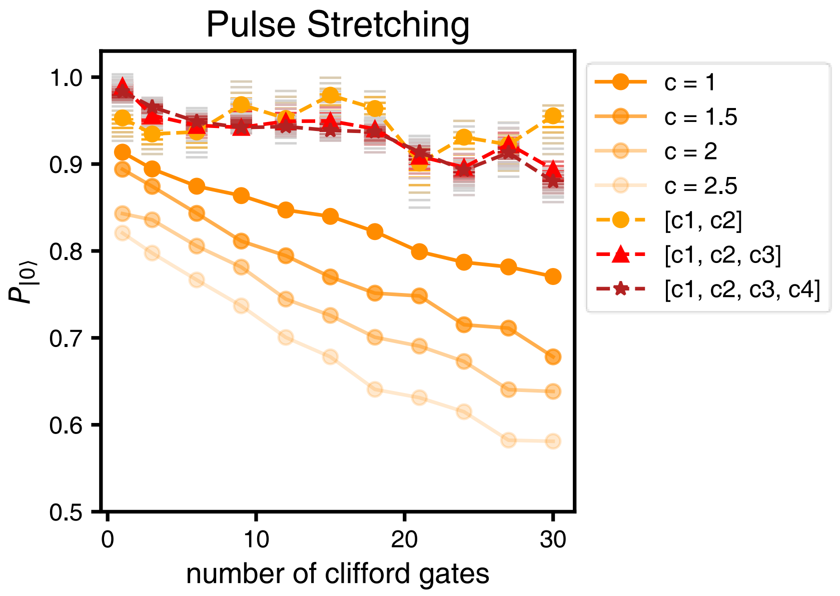
3.1 Results: Randomized benchmarking

Under conditions of time-correlated noise,
Global folding outperforms than Local folding.

Pulse stretching performs well but reveals <u>instability.</u> Indicating <u>the presence of time-correlated noise.</u>

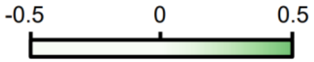


Richardson extrapolation



Linear extrapolation

3.2 Results: Quantum state tomography

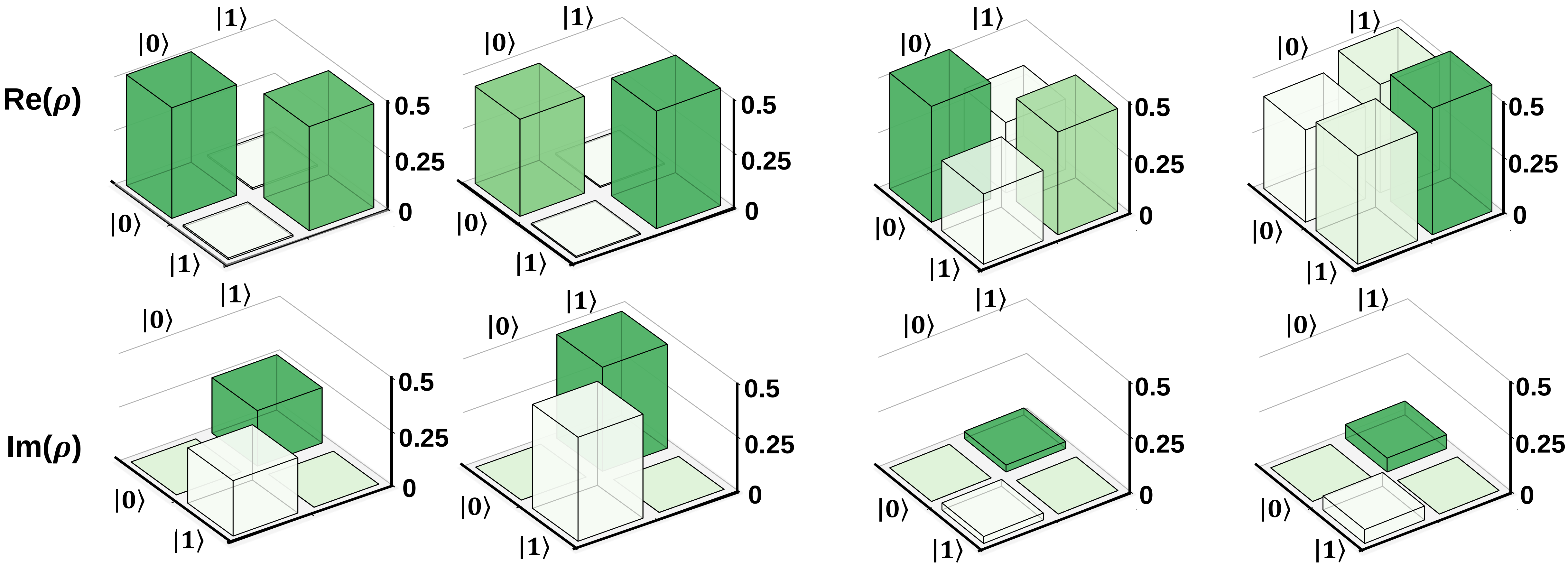


Density matrix of $\frac{X}{2}|0\rangle\ \boldsymbol{state}$

Before mitigation → **After mitigation**

Density matrix of $\frac{Y}{2}|0\rangle\ \boldsymbol{state}$

Before mitigation → **After mitigation**



Fidelity: 0.758

Fidelity: 0.985

Fidelity: 0.822

Fidelity: 0.996

4. Summary

First implementation of zero-noise extrapolation (ZNE) on semiconductor quantum dot

From demonstration of randomized benchmarking,

1. Global folding method outperforms local folding and pulse stretching method.
2. Unitary folding method is a lot more stable than pulse stretching method.

From demonstration of quantum state tomography,

1. By using ZNE and readout error mitigation, we can significantly increase fidelity.
   (From 0.758 to 0.985, 0.822 to 0.996)
2. ZNE is a reliable and relatively simple for mitigating short depth quantum circuit.

5. References

Slides

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Slide 3: Gate operation and measurement of single spin qubit

Slide 4: Gate operation and measurement of single spin qubit

Slide 5: Zero-noise extrapolation: Principle

Slide 6: Zero-noise extrapolation: Optimization

Slide 7: Zero-noise extrapolation: Noise amplification

Slide 8: Zero-noise extrapolation: Noise spectrum

Slide 9: Readout error mitigation

Slide 10: Randomized benchmarking theory

Slide 11: Quantum state tomography theory

Slide 12: Results: Randomized benchmarking

Slide 13: Results: Quantum state tomography

Slide 12: Summary

References

[1] Kandala, A., Temme, K., Córcoles, A.D. *et al.* Error mitigation extends the computational reach of a noisy quantum processor. *Nature* **567**, 491–495 (2019).

[2] E. Magesan, J. M. Gambetta, and J. Emerson, Characterizing quantum gates via randomized benchmarking, Phys. Rev. A 85, 042311 (2012).

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[4] Loss, Daniel, and David P. DiVincenzo., Physical Review A 57, no. 1, 120-26 (1998)

[5] Hicks, R., Kobrin, B., Bauer, C. W. & Nachman, B. et al., Phys. Rev. A 105, 012419 (2022).

[6] Jehyun Kim, Jonginn Yun, Wonjin Jang, Hyeongyu Jang, Jaemin Park, Younguk Song, Min-Kyun Cho, Sangwoo Sim, Hanseo Sohn, Hwanchul Jung, Vladimir Umansky, and Dohun Kim., Phys. Rev. Lett. **129**, 040501

[7] T. Giurgica-Tiron, Y. Hindy, R. LaRose, A. Mari, and W. J. Zeng, in 2020 IEEE International Conference on Quantum Computing and Engineering (QCE) (IEEE, Denver, CO, 2020), p. 306.

Thanks for listening to my presentation!