Intermediate Physics Laboratory 2, Final Project: 2023 Fall

Demonstrating a Chaotic Response of Chua's Circuit and its synchronization using the bidirectional method

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Utilizing a Chua's diode, a non-linear resistor, in conjunction with a gyrator which consists of only resistors, a capacitor, and two op-amps, we successfully constructed a chaotic circuit known as Chua's circuit which is a simple electronic circuit that exhibits classic chaotic behavior. First, we verified whether the circuit exhibits chaotic behavior or not by bifurcating the resistance of the potentiometer. As we change the resistance of a potentiometer, we could see a one-lobe, one-pole, and double-scroll attractor respectively. During the bifurcation process this transition of chaotic behavior was well observed. Moreover, as being chaotic circuit, we could see such transitions in the very tiny region of potentiometer resistance value. Distinctly, a double-scroll attractor with different densities for each lobe was observed. Which indicates that our construction of Chua's circuit and the following bifurcation process were carried out well enough to observe highly subtle transitions between a one-lobe attractor and the double-scroll attractor. Second, we observed a synchronization between two Chua's circuit by connecting a synchronization resistor between two Chua's circuits and adjusting each conditions of two Chua's circuit. Each cases of unsynchronization and synchronization were both observed and the overall behavior and phase of voltages were alike to the that of simulation result.

I. INTRODUCTION

Chua's circuit is a simple electrical circuit that exhibits a classic chaotic behavior. Due to this unique characteristic, Chua's circuit has been an useful tool for understanding chaos theory and numerous research studies have been conducted to explore Chua's circuit and its intriguing characteristics.
In the module 2 and 3, we constructed the Chua's diode and demonstrated the nonlinear I-V characteristic of Chua's diode. In the extension of Module 2 and 3, we constructed Chua's circuit, which consists of Chua's diode, which has an active component, op-amp, and other passive components such as an inductor, capacitors and resistors.
Chua's circuit that we constructed consists of one Chua's diode, two capacitors, one gyrator, and a potentiometer. A gyrator is composed of two operational amplifiers (op-amps) and resistors, and it plays the role of an ideal inductor. In our experiment, two major experiments were done.
First, we verified whether Chua's circuit actually exhibits chaotic behavior or not by bifurcating the resistance of the potentiometer.
Second, we observed a bidirectional synchronization between two Chua's circuits by connecting a synchronization resistor between two Chua's circuits and adjusting each conditions of two Chua's circuit.
Throughout the bifurcation process and the synchronization experiment, we successfully integrated our understanding of a nonlinear components, passive components and functionalities of the digilent Analog Discovery 2 (AD2). These aspects align with what we have learned from Module I to III, signifying the accomplishment of the primary goal for this final module.

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II. BACKGROUNDS

A. Brief Understanding of Chaos Theory

Chaos theory is the scientific field that explain systems that are highly sensitive to its initial conditions. In chaotic system, trivial differences in initial conditions can yield significantly different results. In our experiment, we focused on understanding chaotic attractor. If one attractor has an fractal structure, it is 'strange' attractor. Attractor have the tendency of a system to converge towards a specific state. However, A strange attraction exhibits unpredictable convergence, forming complex patterns over time. One of the chaotic system is the Lorenz system. Lorentz system can be described using ordinary differential equations. The equations that explain Lorentz system is like below. (Eq. 1)

$$\dot{x} = \sigma(y - x) \tag{1a}$$

$$\dot{y} = x(\rho - z) - y \tag{1b}$$

$$\dot{z} = xy - \beta z \tag{1c}$$

$$\sigma, \rho, \beta = \text{coefficient}$$

B. Ideal inductor : Gyrator

Real inductor may deviate from the ideal behavior due to its parasitic effects and other factors. To minimize these differences, a gyrator can be employed. Gyrator is a two-port network element that behaves like an impedance inductor in a electrical circuit. By using gyrator, we can replace a real inductor to an ideal inductor. A typical schematic for a representative gyrator is shown in Fig. 1.

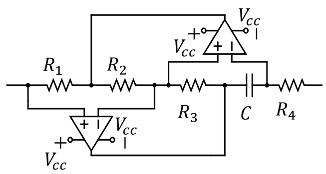


FIG. 1: The schematic of synchronization of representative Gyrator

The inductance of gyrator can be calculated as Eq. 2.

$$L = \frac{R_1 R_3 R_4 C}{R_2} \tag{2}$$

As chaotic behavior of Chua's circuit and its synchronization being so sensitive and requiring hard delicate adjustment, we used the gyrator for synchronization instead of actual inductor. In our expereiment, R_1 = 100 Ω, R_2 = 1.2 kΩ, R_3 = 1 kΩ, R_4 = 2.2 kΩ, C = 100 nF were used to make ideal 18.3 mH inductor.

C. The chaotic system : Chua's circuit

Chua's circuit used in our experiment is constructed by two capacitor, one gyrator, one potentiometer and one Chua's diode. The schematic of Chua's circuit is shown below Fig. 2.

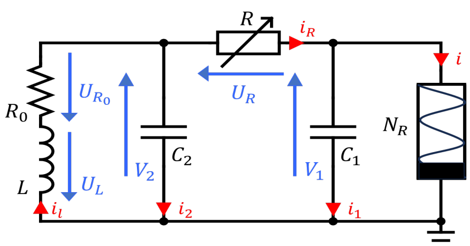


FIG. 2: The schematic of Chua's circuit. N_R is a Chua's diode.

In our experiment, R_0 component was considered as a parasitic resistor. The equations that explain Chua's circuit's system can be obtained by the kirchhoff's law. The equations obtained are like below. G_a and G_b is the conductance that can be calculated from Chua's diode's I-V curve. (Eq. 3)

$$\dot{x} = \alpha(y - x - f(x)) \tag{3a}$$

$$\dot{y} = x - y + z \tag{3b}$$

$$\dot{z} = -\beta y - \gamma z \tag{3c}$$

$$f(x) = bx + \frac{(a-b)(|x+1| - |x-1|)}{2} \tag{3d}$$

$x = V_1/R$, $y = V_2/R$, $z = Ri_L/E$, $\alpha = C_1/C_2$,
$\beta = R^2 C_2/L$, $\gamma = RR_0C_2/L$, $a = RG_a$, $a = RG_b$

In Chua's circuit system, we can observe the diffrent attractors by varying resistance value of R_0. A one-lobe attractor, one-pole attractor, double-scroll attractor and other complex chaotic behavior can be observed.
In Chua's circuit's chaotic system, there is Hopf bifurcation. Hopf bifurcation is a critical point where as a parameter changes, a system's stability switches. In Hopf bifurcation system loses stability and fluctuates. In Chua's circuit's chaotic system, the Hopf bifurcation occur between one-lobe system. As a result, a double-scroll attractor emerges,

D. The bidirectional synchronization between two Chua's circuits

In an AC electric system, synchronization is the process of matching the frequency, phase and voltage of a generator or other source to other electric system.

In Chua's circuit, voltage across capacitor is a distorted AC. Thus, two Chua's circuit can be coupled for synchronization. In our experiment, the bidirectional synchronization was employed for a method for synchronization. In bidirectional synchronization, changes occurred in one system are reflected in the other, and vice versa.

In order to synchronize two Chua's circuits, the same Vcc is applied to Chua's diode's op-amps and the value of components are same. After that, the synchronization resistor is connected between the two circuits. The synchronization resistor is connected to each Chua's circuit's nodes where the C_1 capacitor, variable resistor, and Chua's diode are each connected. (Fig.3)

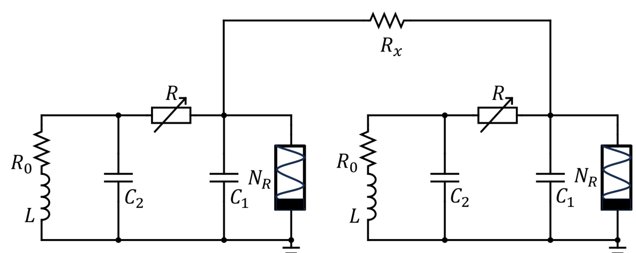


FIG. 3: The schematic of x-coupled bidirectional synchronization of two Chua's circuits

In that cases, it is x-coupled system. In x-coupled system, equation about x_1 and x_2 can be written like below equation.(Eq. 4)

$$\dot{x_1} = \alpha(y_1 - x_1 - f(x_1)) + \delta_x(x_2 - x_1) \tag{4a}$$

$$\dot{x_2} = \alpha(y_2 - x_2 - f(x_2)) + \delta_x(x_1 - x_2) \tag{4b}$$

$$x = V_1/R, y = V_2/R,$$
$$\alpha = C_1/C_2, \delta_x = R\alpha/R_x,$$

To observe synchronization well, synchronization resistor of low resistance should be chosen. It is because coupling strength is stronger when resistor of smaller resistance is used as synchronization resistor. Since synchronization occurs well when coupling strength is stronger, so the resistor of smaller resistance should be used.

III. METHODS

A. Experiment 1: The bifurcation of Chua's circuit

Our experimental setup involved one Analog Discovery 2 (AD2) device, a power supply, one Chua's diode, and one potentionmeter and two capacitors and one Gyrator.
The values of componenets were respectively R_1 = 220 Ω, R_2 = 220 Ω, R_3 = 2.2 kΩ, R_4 = 22 kΩ, R_5 = 22 kΩ, R_6 = 3.3 kΩ. C_1 = 10 nF, C_2 = 100 nF, L = 18.3 mH (Gyrator). R_1 to R_6 is the resistance to make Chua's diode and potentiometer was used as resistor to change the value of resistance.
A 9V Vcc voltage was applied to op-amps Using power supply. The voltages across 10 nF capacitor (V_0) and 100 nF capacitor (V_1) was measured and the plot between V_0 and V_1 was obtained. The obtained results were compared to LTspice simulation results.
The purpose of experiment 1 is to see the bifurcation of Chua's circuit's chaotic system, changing the resistance value using the potentiometer. Using LTspice simulation, the range of resistor values that induce chaotic behavior in an ideal Chua's circuit was calculated. Around this range, The resistance values was varied using potentiometer to observe how the system's behavior will be changed by varying resistance value.

B. Experiment 2: The bidirectional synchronization of two Chua's circuits

To see synchronization of two Chua's circuits, two Chua's circuit were constructed as same in experiment 1. The 100 Ω resistor was used as synchronization resistor, connecting two Chua's circuits.

AD2 channels V_1 and V_2 were connected to 10 nF capacitor of each Chua's circuit and V_1-V_2 curve was plotted. To see synchronization, The values of the potentiometer connected to each Chua's circuit was changed. The obtained results were compared to the LTspice simulation results.

IV. RESULT

A. The chaotic attractor of Chua's circuit

The results of the experiment is in Fig.4. In result, plots that closely resembles the LTspice simulation result were obtained. The one-lobe, one-pole, double scroll attractor was well observed. Moreover, the transition points from one-lobe to double-scroll attractor was observed, therefore, we could observe the Hopf bifurcation of Chua's circuit's system.

However, the experimental results was relatively distorted compared to the simulation result. Additionally, given the nature of chaotic systems, implementing precisely identical conditions to the simulation in experiments is highly challenging, leading to discrepancies between the simulation result and experimental result.

B. Experiment 2: The Synchronization between two Chua's circuits

Both cases of unsynchronization and synchronization in the phase of the two voltage was observed. The experimental results are shown in the figure below. (Fig. 5)
But, we couldn't see the perfect synchronization. The reason is that the V1 voltage's phase wasn't perfectly synchronized with V2 voltages phase as it fluctuates in a short time scale compared to the measurement rate and time.

V. DISCUSSION

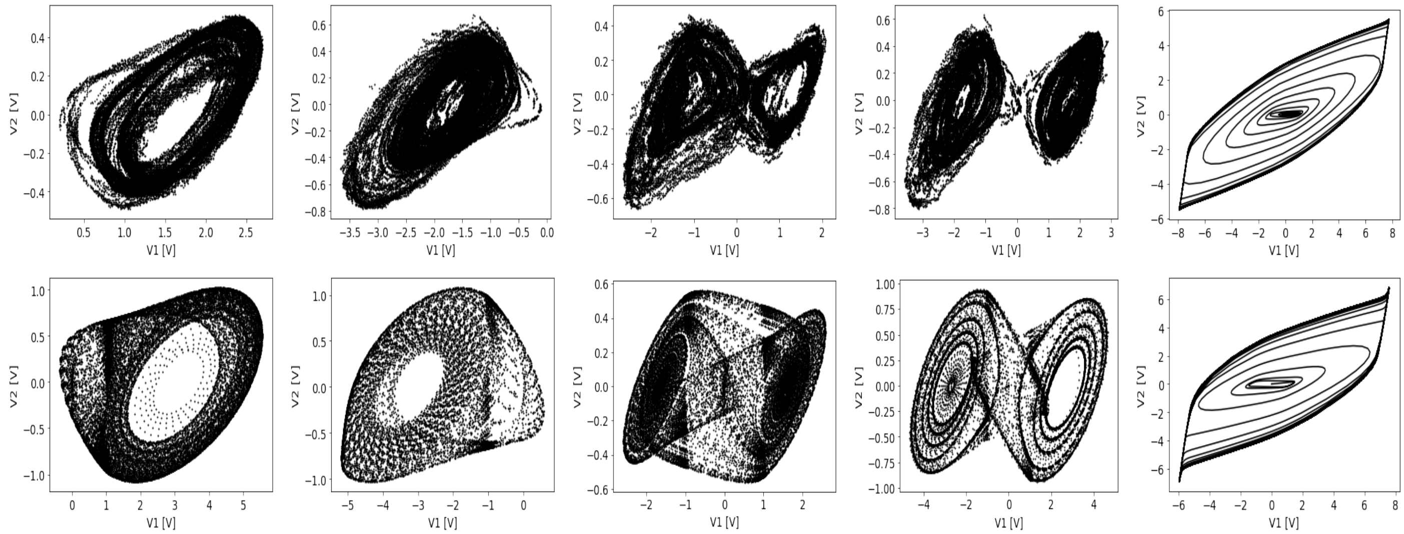
A. Tolerance of electrical component's value and challenges of measurement of resistance

In chaotic system, the small difference of electrical component's value can lead to significant difference. In Experiment 1, even a slight difference in resistance values can cause a transition from one state to other state. In Fig. 4 (a), (b), the difference between the two resistance value is only approximately 3.6 percent, but there was significant differences between their results.

One observation is the asymmetry of two-pole attractor as shown in Fig.4 (c), (d). Simulation result shows fine symmetry in scale of voltage though density can be different along two poles. This may derived from inaccurate value of resistance values used in Chua circuit. While LTspice program utilizes perfect resistance values, our resistors had possibility of having inaccurate values thereby resulting asymmetry.

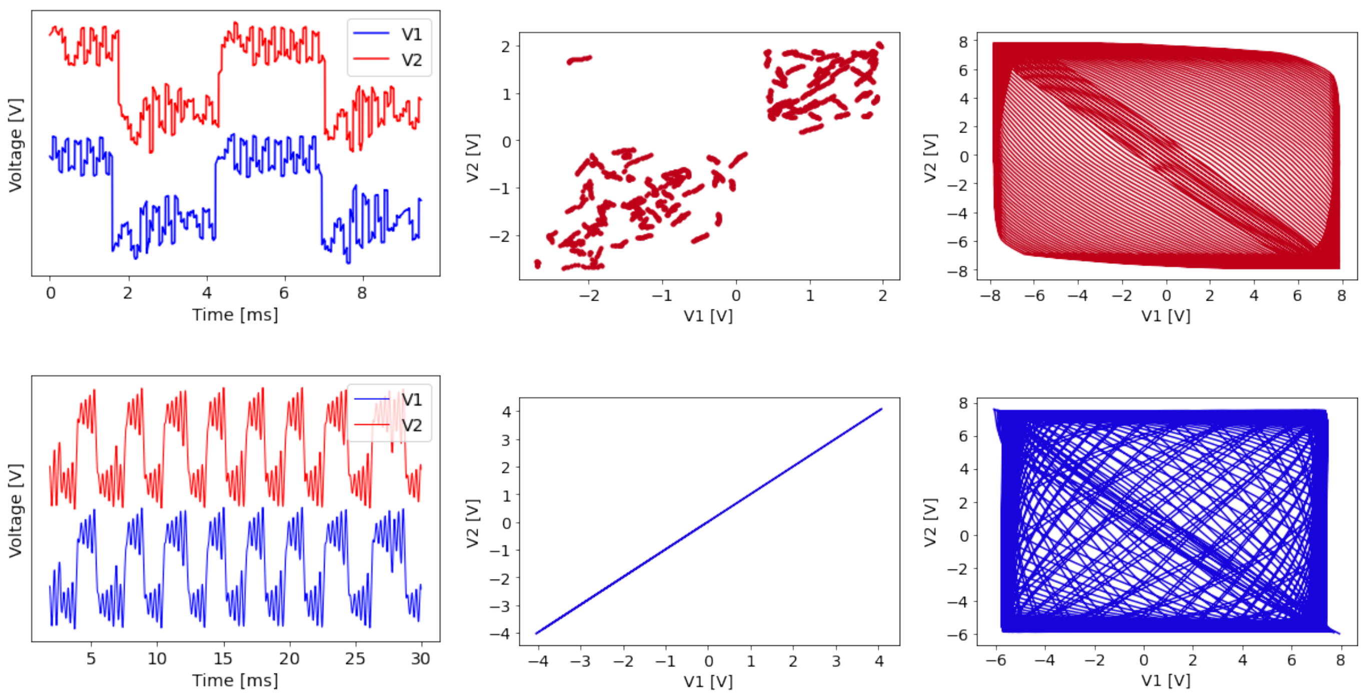
B. The critical point of Chaos attractor observed during the bifurcation process

In the experiment, the attractor that have double-scroll was observed, but there are variations in density around each pole. (Fig.6) The reason why the chaos attractor has different densities around each pole is because double-scroll attractor fluctuates between the left lobe and the right lobe. Therefore, depending on the potentiometer's resistance values, Chua's circuit's initial conditions, and the measurement moment, the double-scroll attractor's density can be different. A more precise observation and calculation of this phenomenon are needed for a deeper understanding of chaos systems.



(a) R = 1.92 kΩ (b) R = 1.927 kΩ (c) R = 1.6 kΩ (d) R = 1.85 kΩ (e) R = 1.4 kΩ

FIG. 4: Hopf Bifurcation of system of Chua's circuit (Top: Experiment, Bottom: LTspice Simulation)



(a) synchronized time-V_1,V_2 plot (b) synchronized V_1-V_2 plot (c) Unsynchronized V_1-V_2 plot

FIG. 5: Synchronization of two Chua's circuits(Top:Experiment, Bottom:Simulation)

C. Proper choice of synchronization resistor

In LTspice simulation, we could see non-synchronization with high resistance but it was difficult to see synchronization. But when we adjust the resistance value lower, we could see synchronization. This is natural behavior, given that high resistance means two circuit are separated. We could check the reasons in the Eleonora Bilotta's experiment[1]. They show that when synchronization resistor's resistance value is low, the coupling strength is stronger. So, regardless of the initial conditions of the two Chua's circuits (chaotic-chaotic, chaotic-

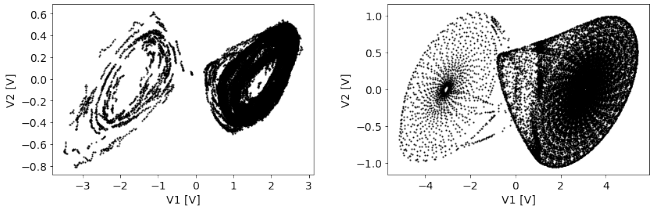


FIG. 6: Density of chaos attractor: (a) experiment (b) simulation

damped), we can see the non-chaotic synchronization. So, to see perfect synchronization between phases between two voltages, using a resistor with low resistance is proper. so, we used a 100Ω resistor for the reasons explained earlier.

However, as you can see in Fig.5-(b), it showed a lot of error, and there is discontinuity around $V_1 = 0.5$ [V]. We could say that this is not entirely due to the resistance value. It is because perfect synchronization can be detected between two identical Chua's circuit. However, because of component's tolerance and characteristic of chaotic behavior, to make two Chua's circuit identical is daunting. Also consumption of heat during synchronization process is inevitable. Accordingly, research using other electric components like capacitors, op-amps was done. As shown in the experiment of Chua, et. al., they could see synchronization with additional op-amp by making master-slave structure.[2] So, with further component, we need to make more delicate circuit and need further study.

Furthermore, in the Eleonora Bilotta's experiment[1], they showed another effect of synchronization resistor - the phase difference and its transition. They showed that when the synchronization resistor's resistance value get larger the phase transition occur. So when bifurcating synchronization resistor, a difference in the voltage phases between two Chua's circuits emerges.
Also, as the synchronization resistor's resistance gets larger, the correlation between amplitude of voltages of two Chua's circuit gets weaker. Hence, when we use larger resistance value for the synchronization resistor, we can see the complex phase diagrams and transitions of phase diagrams. Thus, connecting a potentiometer between the two Chua's circuits and changing the resistance would allow us to observe phase transitions. However, synchronization between the two Chua's circuits can be significantly influenced by even minor errors. Therefore, further circuit optimization such as soldering and using high-quality circuit elements is needed.

VI. CONCLUSION

In conclusion, we could explore and elaborate upon the chaotic behavior of Chua's circuit and its bidirectional synchronization.

Before the experiment, we constructed a gyrator to implement an ideal inductor and employed this gyrator in our Chua's circuit. Utilizing the gyrator allowed us to observe more stable, accurate chaotic behavior of Chua's circuit, enhancing the precision of our experiment.

In experiment 1, We observed the transition from one pole attractor to double scroll attractor by bifurcating potentiometer's resistance value. Moreover, a fascinating critical point during bifurcation is observed, indicating not only the bifurcation process is done precisely but also that Chua's circuit is precisely constructed. Still, there exists a slight difference between the experiment results and LTspice simulation. We concluded that this discrepancy occured due to tolerances in component values and fluctuations occurring in real-time experiments with actual circuit elements.

In experiment 2, the unsynchronized case was well observed, but the measurement of perfectly synchronized cases was challenging. This is due to the nature of a chaotic system and further optimization is needed such as soldering and using high-quality circuit elements.

We discussed 3 things in discussion part: 1. Tolerance of electrical components, 2. the critical point of chaos attractor, and 3. proper choice of the synchronization resistor and its bifurcation. Especially, we discussed why the resistor with small resistance value should be used, in terms of coupling strength.

To summarize, the experiments provided valuable result that explains chaotic behavior of Chua's circuit and its bidirectional synchronization. Similarity between experimental results and simulation results reinforces the credibility of our findings. That is, we successfully built Chua's circuit and conducted crucial measurements, including the transition of chaotic voltage phases and achieving synchronization.

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