Bosonic Error Correction

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CONTENTS

INTRODUCTION

Quantum Error Correction (QEC)

Physical quantum system		Logical qubit state
$\|00000\rangle$	$\xrightarrow{\text{Encoding}}$	$\|0_L\rangle$
$\|11111\rangle$		$\|1_L\rangle$

Define error operator $\hat{E}_i$ (e.g. $\hat{\sigma}_-, \hat{\sigma}_z, \hat{a}, \hat{a}^2 \cdots$)

Code space: Hilbert space of logical qubit states $|0_L\rangle, |1_L\rangle \in H_{code}$

Error space: Hilbert space of errored states $E_i|0_L\rangle, E_j|1_L\rangle, \cdots \in H_{error}$

Redundancy matters

Ref: Michael A. Nielsen, Isaac L. Chuang. *Quantum Computation and Quantum Information, CH10* (Cambridge University Press, 2010)

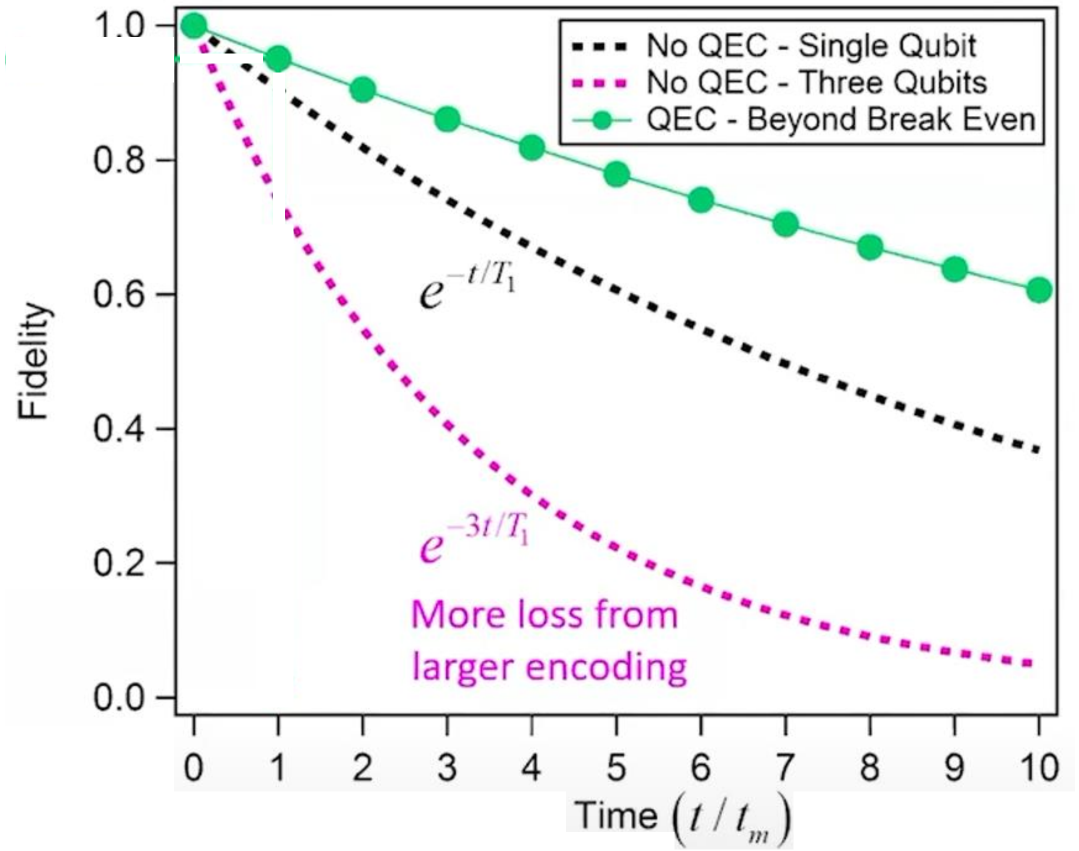
INTRODUCTION

<u>**Quantum error correction is possible only if**</u>

1. Hilbert space of the system must be large that $H_{code}, H_{error} \subseteq H_{sys}$
2. Knill-Laflamme QEC condition

   $\langle a_L | E_i^\dagger E_j | b_L \rangle = \alpha_{ij}\delta_{ab}$

→ Error must not mix different logical states.

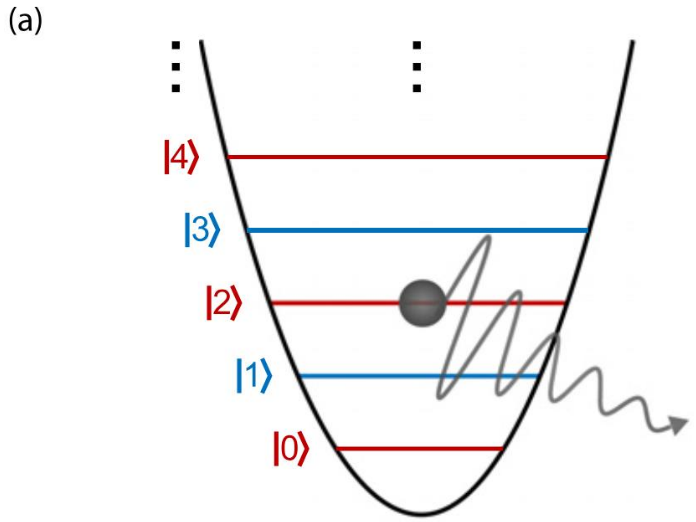


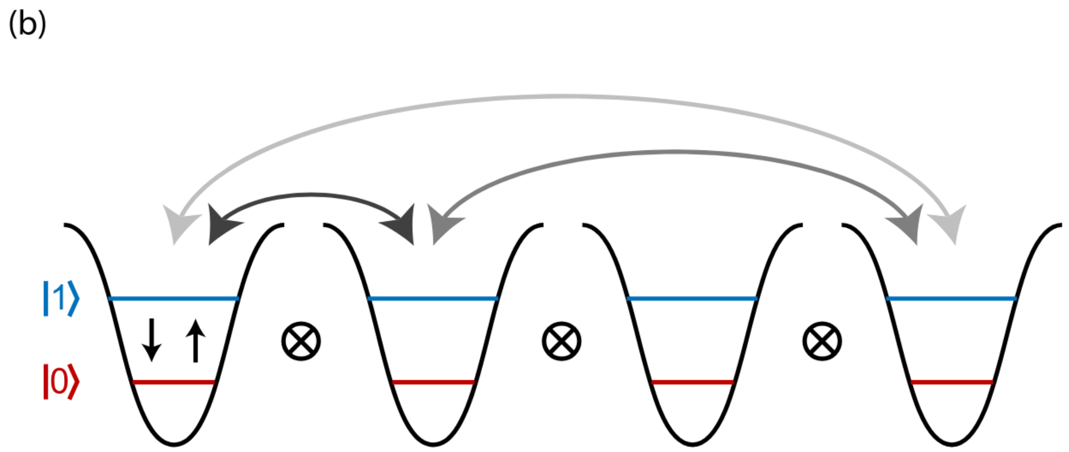
Beyond Break Even point is important in QEC.

Ref: Michael A. Nielsen, Isaac L. Chuang. *Quantum Computation and Quantum Information, CH10* (Cambridge University Press, 2010)

INTRODUCTION

Bosonic Code **vs** **Multi-qubit Code**





Many levels in single mode.

Excitation & dephasing loss: $\hat{a}, \hat{a}\hat{a}^{\dagger}$

Single mode is sufficient for encoding logical state!

Easy to realize highly coherent cavity and a coupling between cavity and ***superconducting qubits.***

Tensor product of qubits.

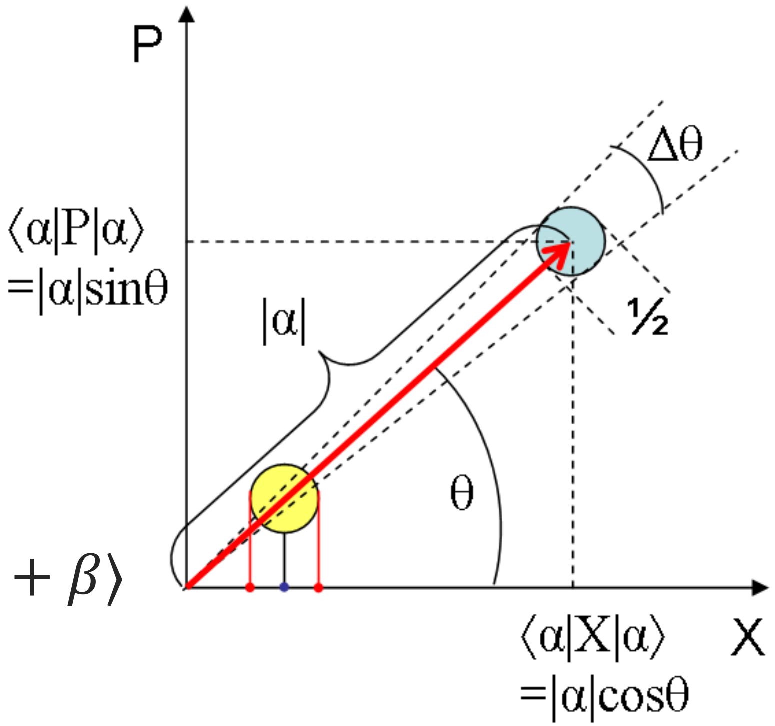
Independent Pauli errors: $\{\hat{I}, \hat{\sigma}_{\mathrm{x}}, \hat{\sigma}_{\mathrm{y}}, \hat{\sigma}_{\mathrm{z}}\}^{\otimes N}$

Requires multi-qubits for encoding logical state.

BASICS OF BOSONIC SYSTEM

Coherent State

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$



Properties of coherent state

1. For displacement operator $D(\alpha)$, $D(\alpha)|0\rangle = |\alpha\rangle$, $D(\alpha)|\beta\rangle \propto |\alpha + \beta\rangle$
2. Eigenstate of photon annihilation operator: $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$
3. **Minimum uncertainty**
4. **Quasi-classical state**

J-P. Gazeau. *Coherent States in Quantum Physics,* Wiley-VCH, Berlin, 2009.

BASICS OF BOSONIC SYSTEM

Wigner Quasi-Probability distribution

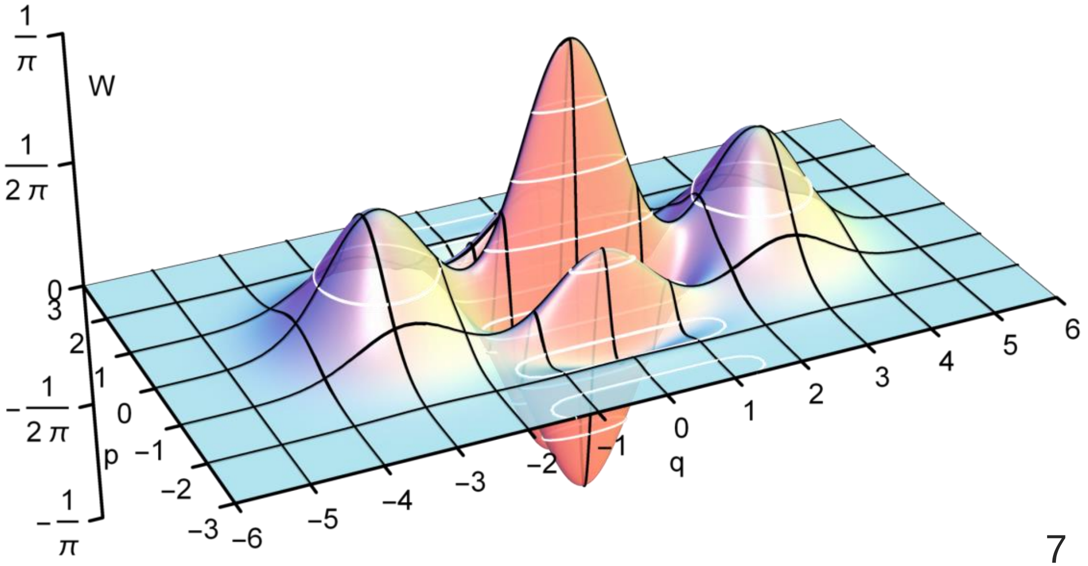
$$W(x,p) \equiv \frac{1}{\pi\hbar}\int_{\infty}^{\infty} dy\, \psi^*(x+y)\psi(x-y)e^{\frac{2ipy}{\hbar}}$$

Not a probability distribution function.

(no sigma additivity, $W \in \mathbb{R}$ (can have negative value, non-classical)

<u>Probability distribution of cavity state in phase space.</u>

Wigner distribution of a Cat state



Decoherence

$$L(\rho) = \Gamma\left(L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}\right)$$

1. Photon loss: $\{\hat{a}\}$
2. Dephasing: $\{\hat{a}^\dagger\hat{a}\}$
3. Thermal noise:$\{\hat{a}, \hat{a}^\dagger\}$
   from random displacements $D(\alpha)$

E. P. Wigner (1932). "On the quantum correction for thermodynamic equilibrium". *Physical Review*. **40** (5): 749–759.

BOSONIC ERROR CORRECTION

1. Cat Code

Simplest encoding: $|0_L\rangle = |\alpha\rangle$, $|1_L\rangle = |-\alpha\rangle \rightarrow$ not error correctable since $H_{code} = H_{error}$ under photon loss error $\hat{a}$.

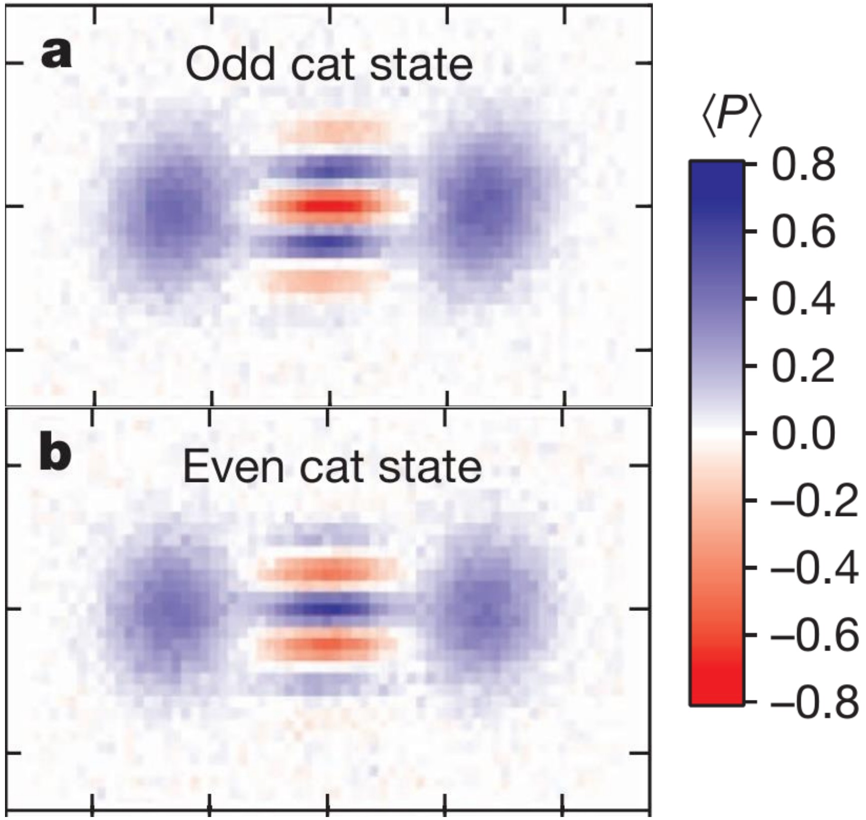
Thus, alternatively we choose cat states, <u>Two cat states.</u>

$$|0_L\rangle = |C_\alpha^+\rangle \equiv \frac{|\alpha\rangle+|-\alpha\rangle}{\sqrt{2\left(1+e^{-|\alpha|^2}\right)}},\ |1_L\rangle = |C_{i\alpha}^+\rangle \equiv \frac{|i\alpha\rangle+|-i\alpha\rangle}{\sqrt{2\left(1+e^{-|\alpha|^2}\right)}}$$

$$|0_E\rangle = |C_\alpha^-\rangle \equiv \frac{|\alpha\rangle-|-\alpha\rangle}{\sqrt{2\left(1-e^{-|\alpha|^2}\right)}},\ |1_E\rangle = |C_{i\alpha}^+\rangle \equiv \frac{|i\alpha\rangle-|-i\alpha\rangle}{\sqrt{2\left(1-e^{-|\alpha|^2}\right)}}$$

Orthogonal for large $|\alpha|^2 = \bar{n} \geq 2$.

Protection against photon loss $\epsilon = \{1, a\}$.



Deterministic decay: $\alpha(t) = \alpha e^{-k_s t/2} \rightarrow$ periodic repumping is needed.

Sun, Luyan, et al. "Tracking photon jumps with repeated quantum non-demolition parity measurements." *Nature* 511.7510 (2014): 444-448.

BOSONIC ERROR CORRECTION

1. Cat Code

$$|0_L\rangle = |C_\alpha^+\rangle,\ |1_L\rangle = |C_{i\alpha}^+\rangle \text{ and } |0_E\rangle = |C_\alpha^-\rangle,\ |1_E\rangle = |C_{i\alpha}^-\rangle$$

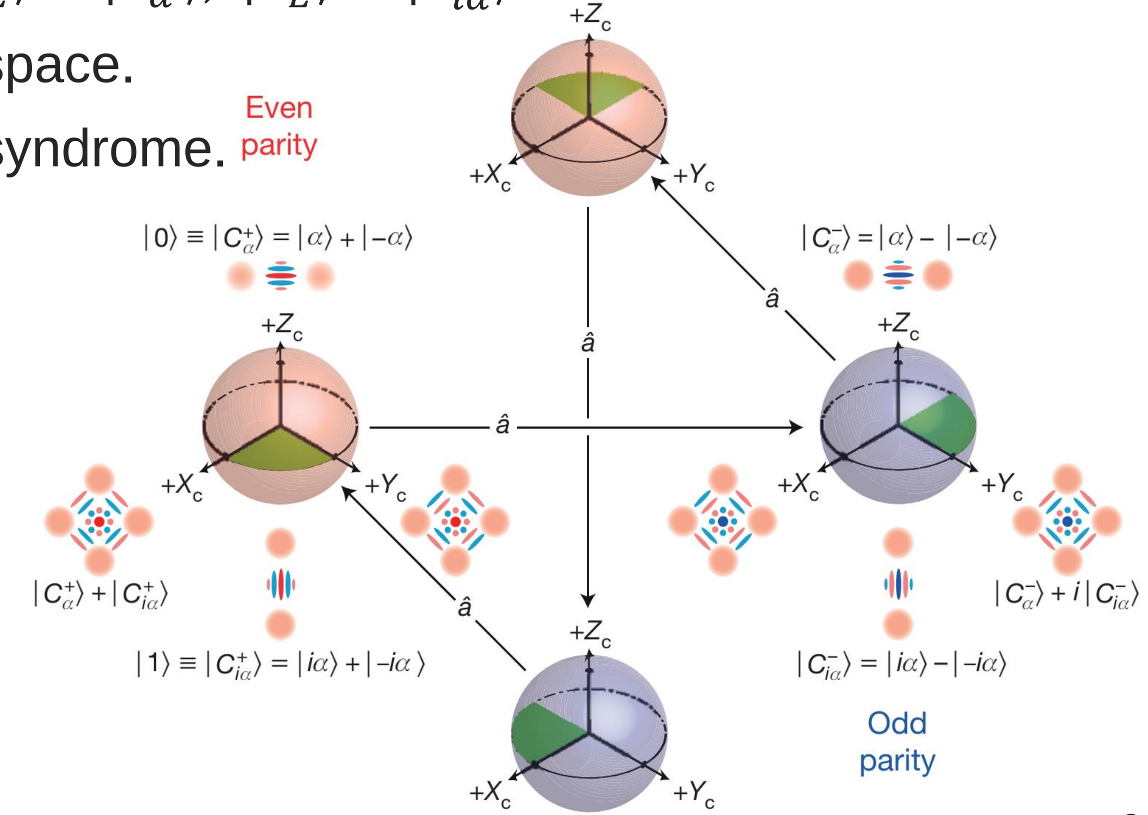
Even parity for code space and odd parity for error space.

→ Parity measurement can be used to detect error syndrome.

Code space states has four-fold degeneracy.

: Count the number of errors and apply single correction gate at the last stage can fully remove the effect of error

→ High QEC fidelity



Ofek, Nissim, et al. "Extending the lifetime of a quantum bit with error correction in superconducting circuits." *Nature* 536.7617 (2016): 441-445.

BOSONIC ERROR CORRECTION

Physical Implementation of Cat Code

1. Encoding

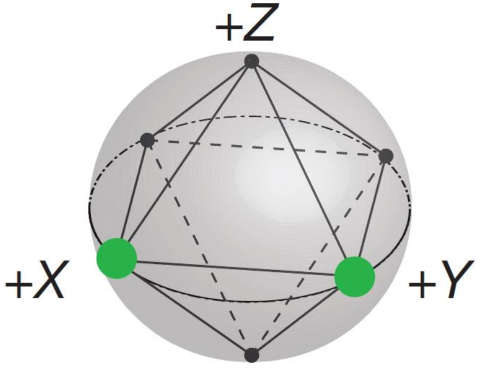
$$(a|g\rangle + b|e\rangle) \otimes |0\rangle \rightarrow |g\rangle \otimes (a|C_\alpha^+\rangle + b|C_{i\alpha}^+\rangle)$$

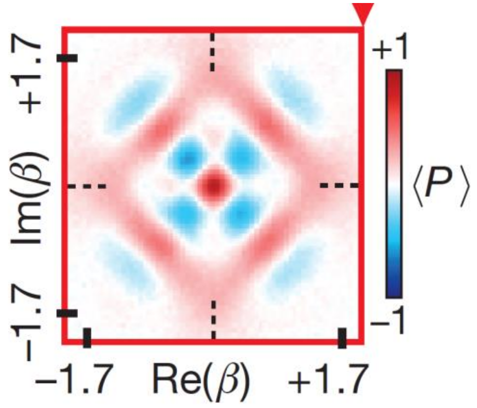
Cat states are too complicate to encode using analytic methods.

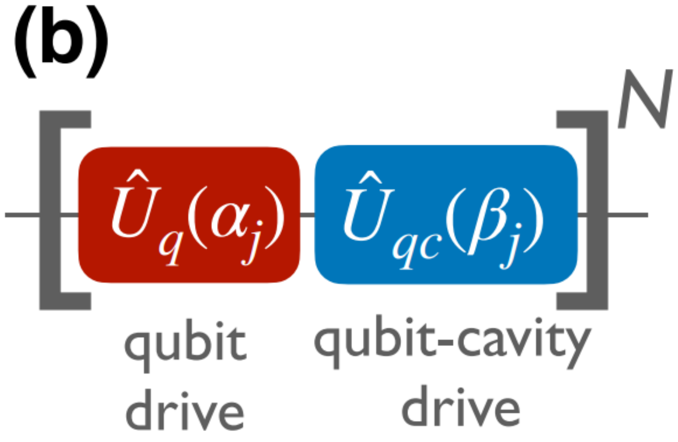
Thus, we use technique called GRAPE (Gradient Ascent Pulse Engineering).

Using gradient ascent algorithm which is machine learning tech,

find proper parameters of unitary gates for encoding.







[1] Ofek, Nissim, et al. "Extending the lifetime of a quantum bit with error correction in superconducting circuits." *Nature* 536.7617 (2016): 441-445., [2] *PRX Quantum* 4.3 (2023): 030305.

BOSONIC ERROR CORRECTION

Physical Implementation of Cat Code

2. Track Error Syndrome

Use QND parity measurement using transmon ancilla.

$$|\psi\rangle = |g\rangle \frac{|\alpha\rangle \pm |-\alpha\rangle}{\sqrt{2}}$$

$$\downarrow Y_{\pi/2}$$

$$|\psi\rangle = \frac{|g\rangle + |e\rangle}{\sqrt{2}} \frac{|\alpha\rangle \pm |-\alpha\rangle}{\sqrt{2}}$$

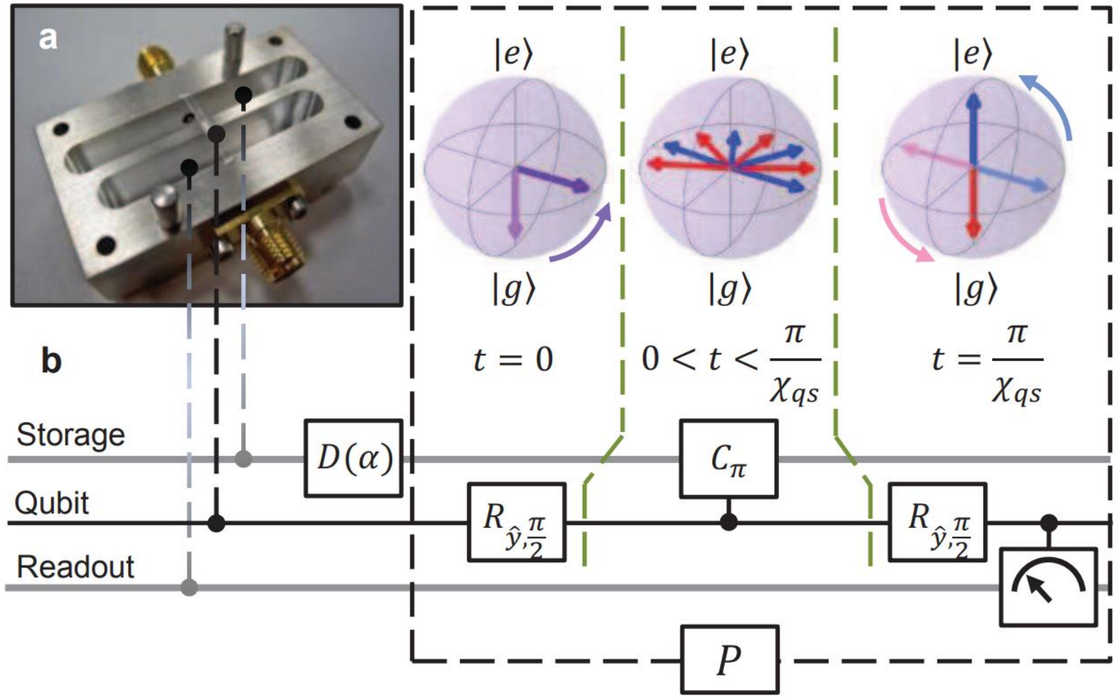
$$\downarrow U(\Delta t = \pi/\chi) \text{ by } H_{int} = -\chi a^\dagger a |e\rangle\langle e| \qquad \chi \sim 1.78 \text{ MHz}$$

$$|\psi\rangle = \frac{1}{2}|g\rangle(|\alpha\rangle \pm |-\alpha\rangle) + \frac{1}{2}|e\rangle(|\alpha e^{i\pi}\rangle \pm |-\alpha e^{i\pi}\rangle)$$

$$= \frac{|g\rangle \pm |e\rangle}{\sqrt{2}} \frac{|\alpha\rangle \pm |-\alpha\rangle}{\sqrt{2}}$$

$$\downarrow Y_{-\pi/2}$$

$$|\psi\rangle = |g\rangle \frac{|\alpha\rangle + |-\alpha\rangle}{\sqrt{2}} \text{ or } |e\rangle \frac{|\alpha\rangle - |-\alpha\rangle}{\sqrt{2}}$$



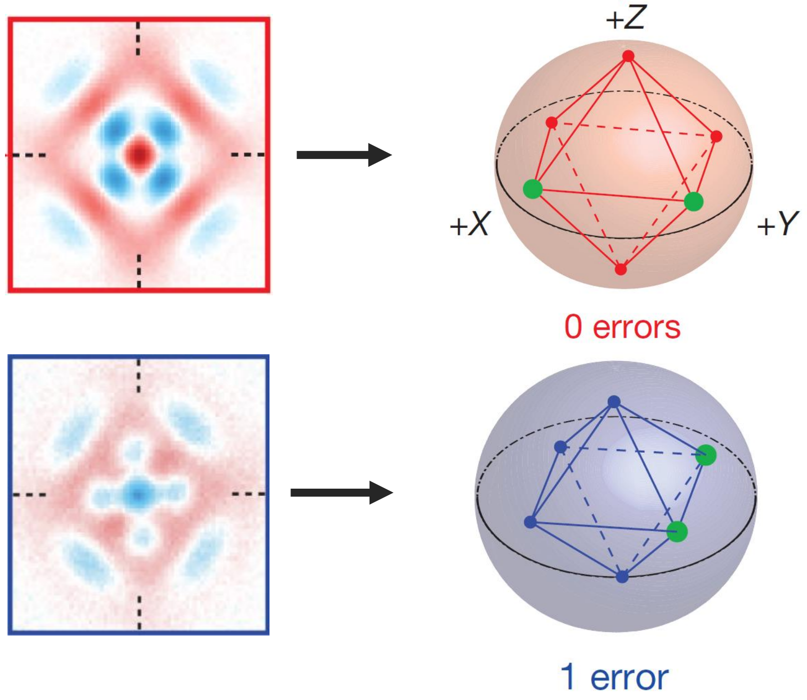
Sun, Luyan, et al. "Tracking photon jumps with repeated quantum non-demolition parity measurements." *Nature* 511.7510 (2014): 444-448.

BOSONIC ERROR CORRECTION

Physical Implementation of Cat Code

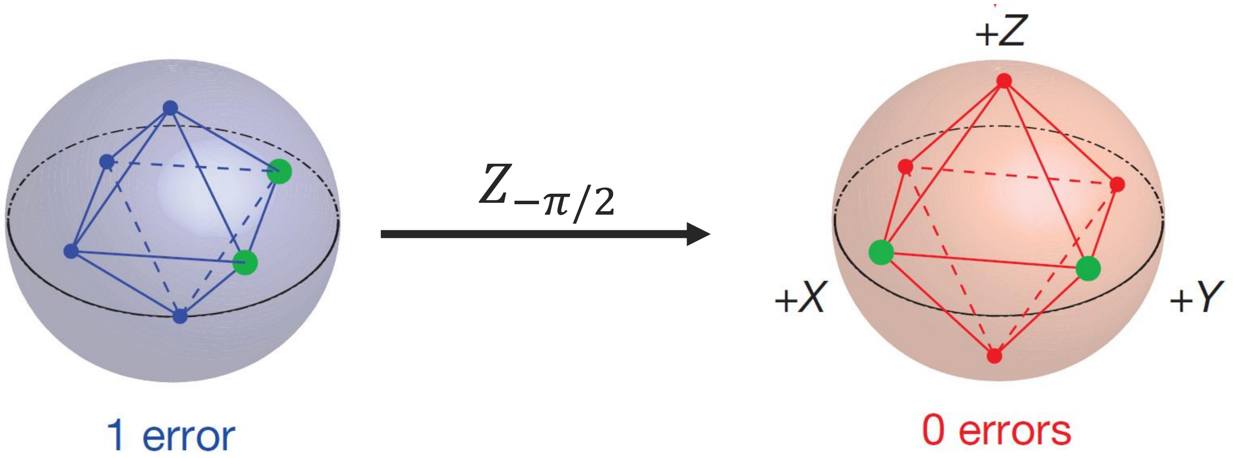
3. <u>Decoding</u>

Reverse sequence of encoding.



4. <u>Correction Gate</u>

Depending on the number of error counted in 'tracking error syndrome' stage, apply proper correction gate to recover original state



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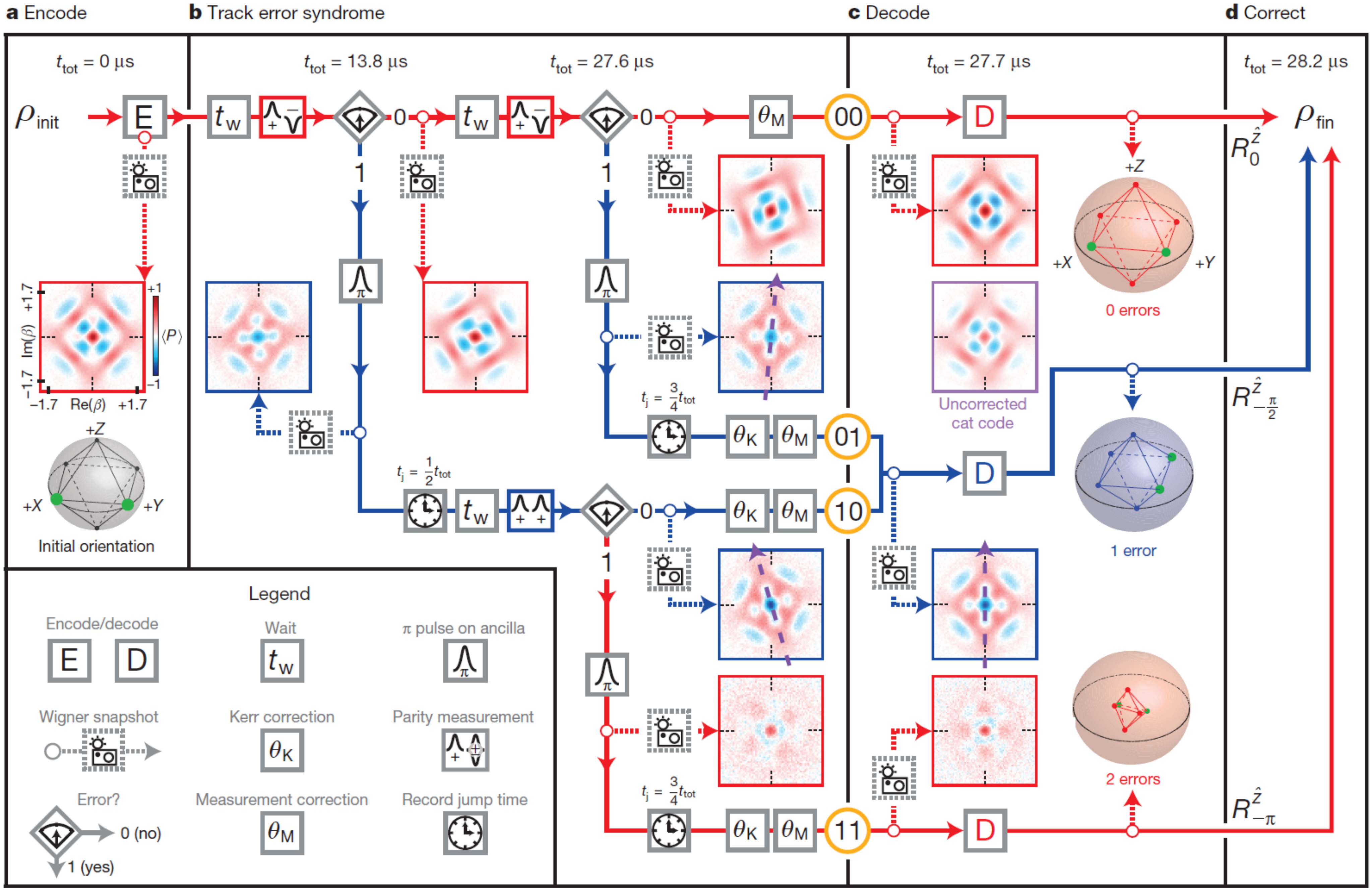
Total



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BOSONIC ERROR CORRECTION

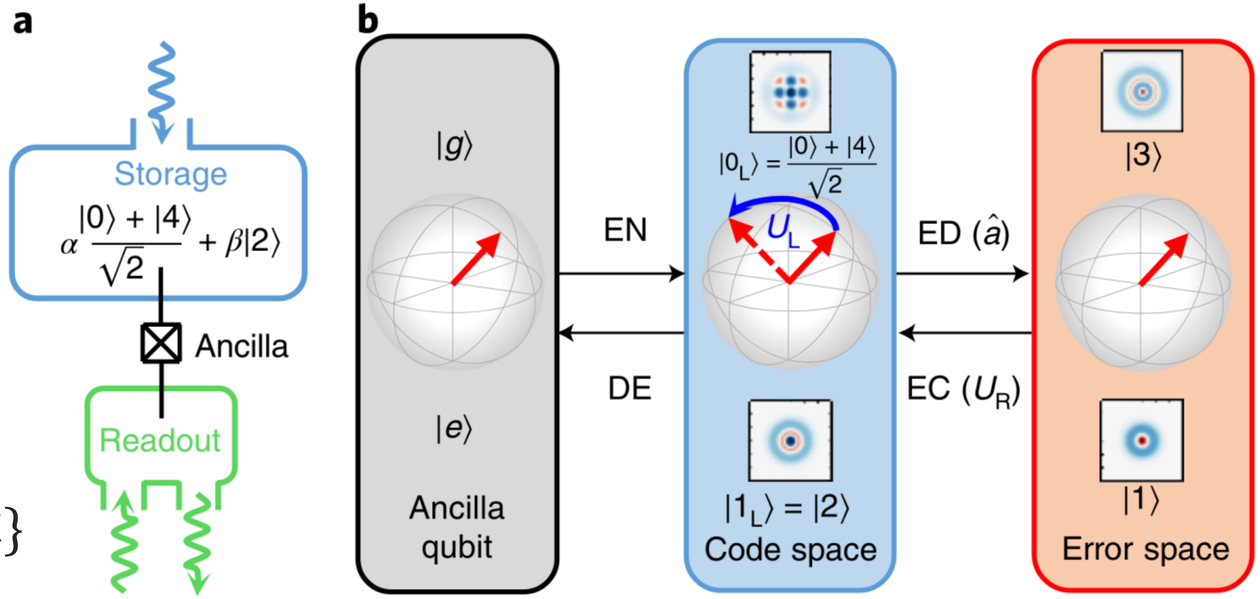
2. Binomial Code

Simplest binomial → **Kitten code**

$$|0_L\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |4\rangle), |1_L\rangle = |2\rangle$$

Mean photon number = 2 : Satisfies QEC cond.

However, it is not protected under $\hat{a}^2$, $\epsilon = \{1, \hat{a}\}$



Instead, we can consider $|0_L\rangle = \frac{|0\rangle+\sqrt{3}|6\rangle}{2}$, $|1_L\rangle = \frac{\sqrt{3}|3\rangle+|9\rangle}{2}$ → <u>Higher-order binomial code</u>

Initial Photon number is 9/2, and after applying $\hat{a}$, it is 6.

Even after applying $\hat{a}$, mean photon numbers are equal thus QEC cond. is satisfied.
Therefore, it is protected up to $\hat{a}^2$ $\epsilon = \{1, \hat{a}, \hat{a}^2, \hat{n}\}$

Cai, Weizhou, et al. "Bosonic quantum error correction codes in superconducting quantum circuits." *Fundamental Research* 1.1 (2021): 50-67.

BOSONIC ERROR CORRECTION

2. Binomial Code

In this way, we can make generalized code protected up to arbitrary orders of creation ($\hat{a}^\dagger$), annihilation ($\hat{a}$), and dephasing error ($\hat{n}$).

$$|0_L\rangle = \frac{1}{\sqrt{2^N}} \sum_{p,even}^{[0,N+1]} \sqrt{C_{N+1}^{p}} |p(S+1)\rangle$$

$$|1_L\rangle = \frac{1}{\sqrt{2^N}} \sum_{p,even}^{[0,N+1]} \sqrt{C_{N+1}^{p}} |p(S+1)\rangle$$

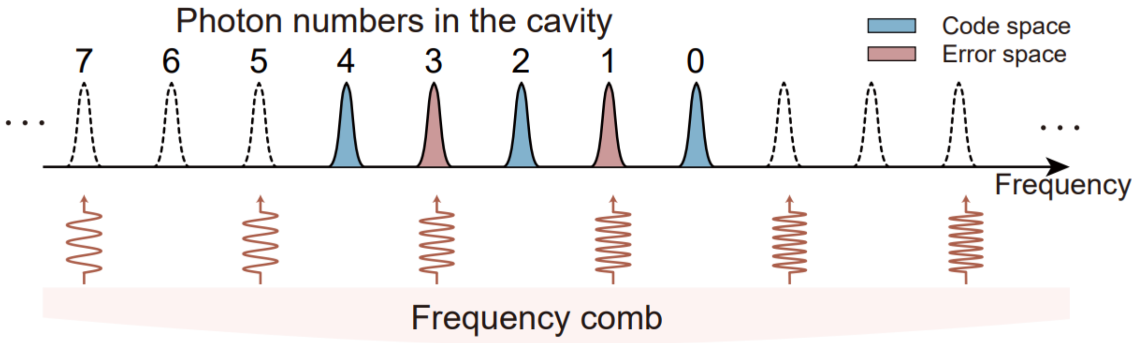
$$\epsilon = \left\{1, \hat{a}, \hat{a}^2, \cdots, \hat{a}^L, \hat{a}^\dagger, {\hat{a}^\dagger}^2, \cdots, \left(\hat{a}^\dagger\right)^G, \hat{n}, \hat{n}^2, \cdots, \hat{n}^D\right\}$$

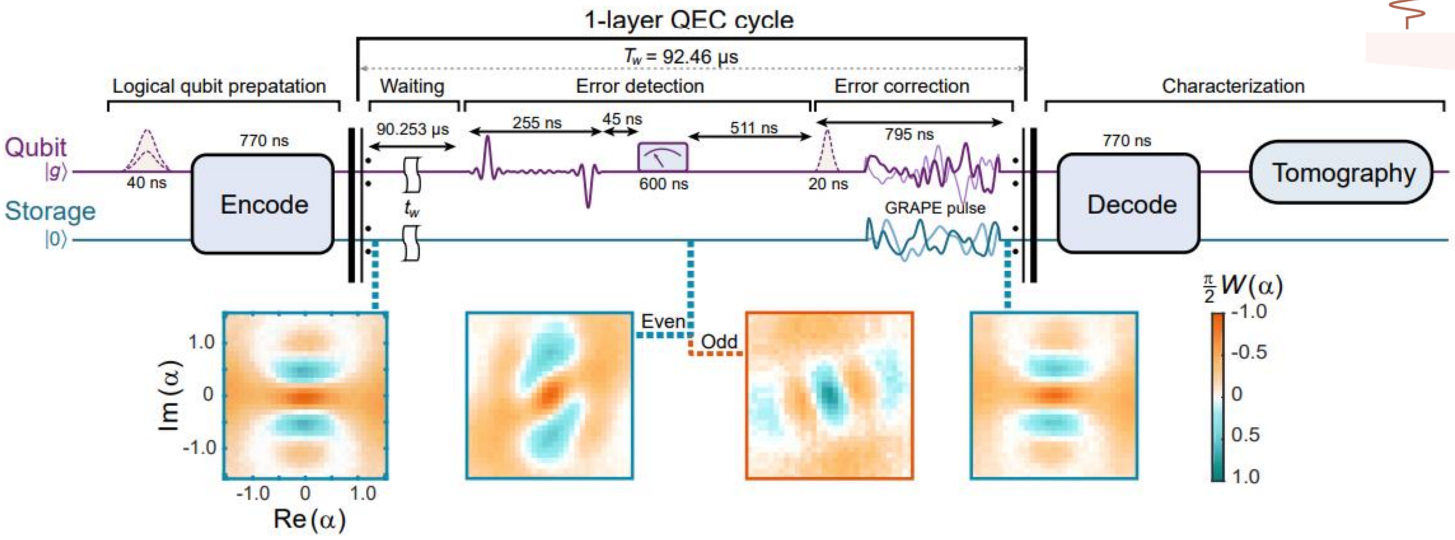
Exactly corrects errors up to polynomial orders.

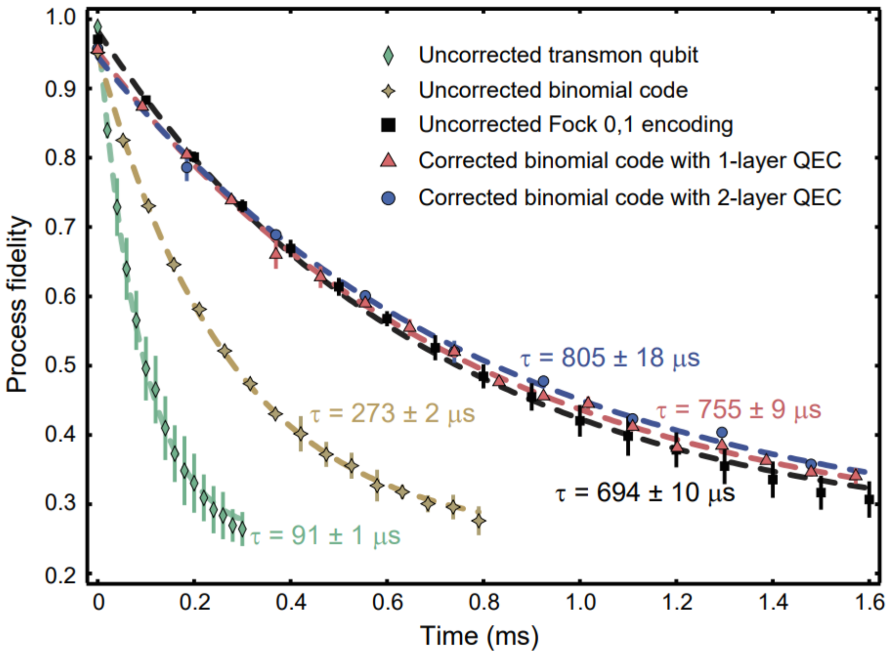
Cai, Weizhou, et al. "Bosonic quantum error correction codes in superconducting quantum circuits." *Fundamental Research* 1.1 (2021): 50-67.

BOSONIC ERROR CORRECTION

2. Binomial Code - Experiments







Kitten code with superconducting device

Reached the break-even

Ni, Zhongchu, et al. "Beating the break-even point with a discrete-variable-encoded logical qubit." *Nature* 616.7955 (2023): 56-60.

BOSONIC ERROR CORRECTION

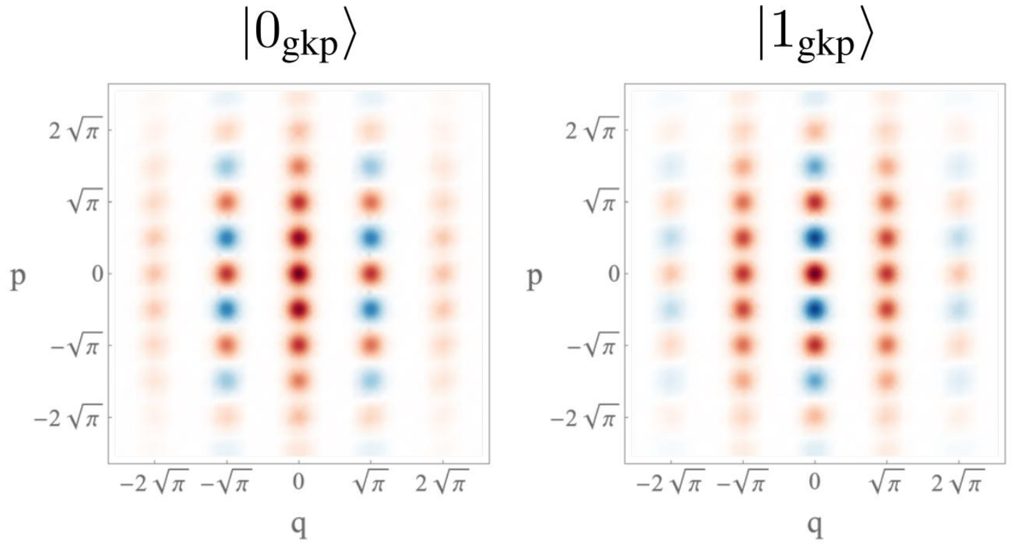
3. GKP Code

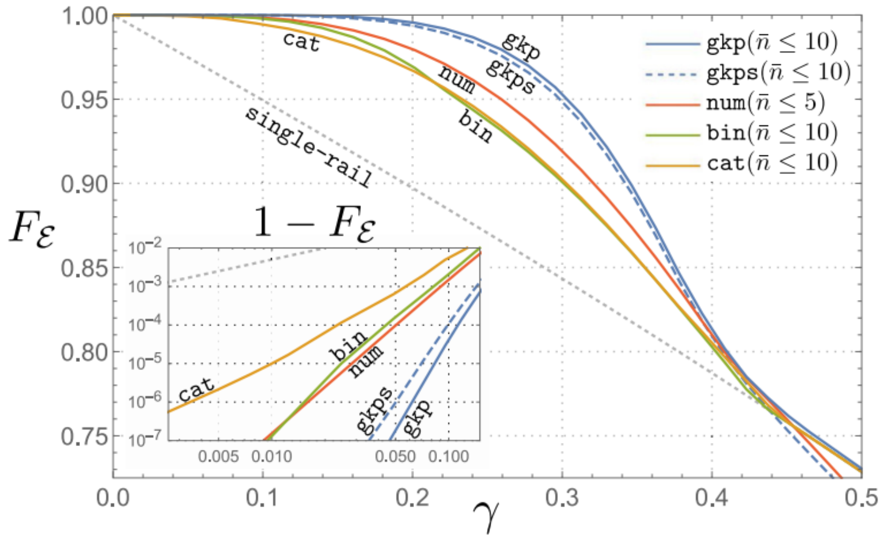
Lattice of squeezed or coherent states.

Displacement errors can be detected & corrected.

$$|0_L\rangle = \sum_{-\infty}^{\infty} \left|q = 2s\sqrt{\pi}\right\rangle, |1_L\rangle = \sum_{-\infty}^{\infty} \left|q = (2s+1)\sqrt{\pi}\right\rangle$$

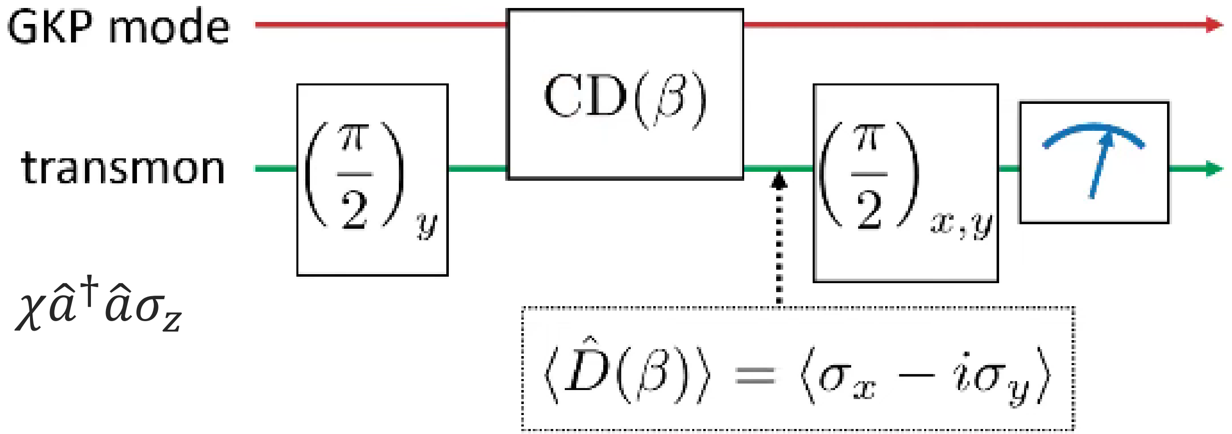
Strongest code using infinite number of photons.





Noh, Kyungjoo, and Christopher Chamberland. "Fault-tolerant bosonic quantum error correction with the surface–Gottesman-Kitaev-Preskill code." *Physical Review A* 101.1 (2020): 012316.

BOSONIC ERROR CORRECTION

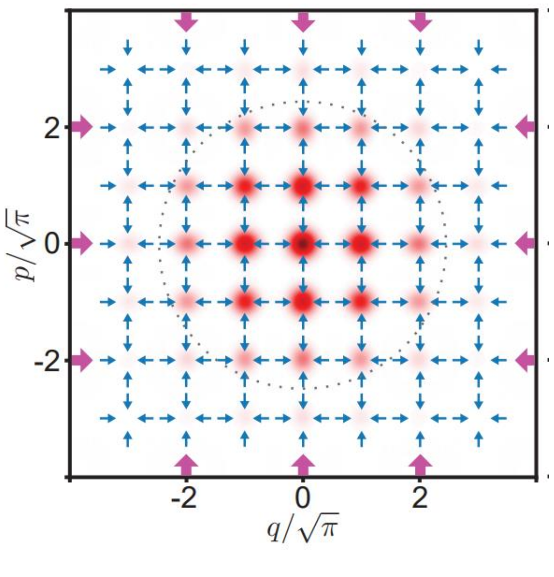


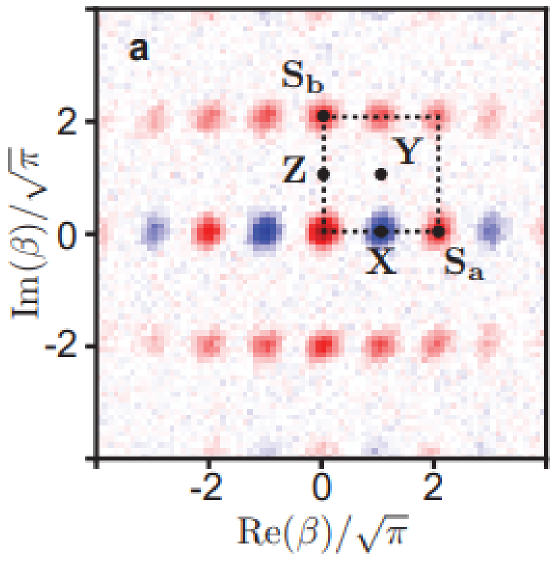
3. GKP Code

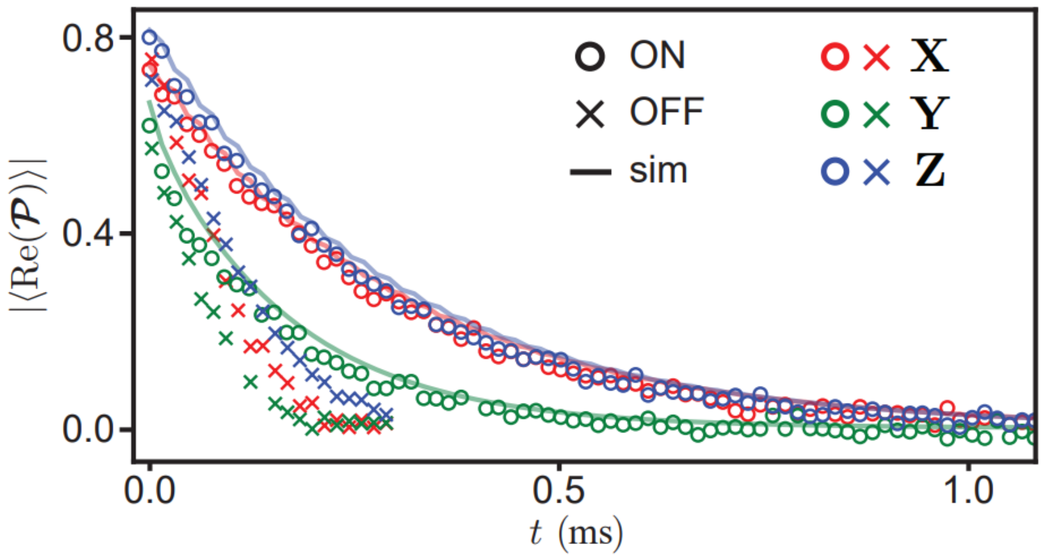
Lattice: from driving a cavity

$$H \propto i\varepsilon\hat{a}^\dagger - i\varepsilon^*\hat{a} + \chi\hat{a}^\dagger\hat{a}\sigma_z \propto \left(\alpha\hat{a}^\dagger + \alpha^*\hat{a}\right)\sigma_z + \chi\hat{a}^\dagger\hat{a}\sigma_z$$

for $\hat{a} \rightarrow \hat{a} + \alpha$







Ni, Zhongchu, et al. "Beating the break-even point with a discrete-variable-encoded logical qubit." *Nature* 616.7955 (2023): 56-60.

BOSONIC ERROR CORRECTION

	Cat Code	Binomial Code	GKP Code
Encoding	Coherent states	Fock states	Lattice of Squeezed states
Error Set	Photon loss: $\{I, a\}$	$\{I, \varepsilon, \ldots, \varepsilon^n\}$ $for\ \varepsilon = a, a^\dagger, a^\dagger a$	Small displacements from all errors
Advantages	Only need to track the number of jumps	Can correct arbitrary orders of errors	Optimal bosonic code (Highest performance among three)
Limitations	Limited correctable errors	Need many photons to correct more errors	Complex code structure → hard to create
Platforms	SC circuits, Ions	SC circuits, Ions, optics	SC circuits

Albert, Victor V., et al. "Performance and structure of single-mode bosonic codes." *Physical Review A* 97.3 (2018): 032346.

BOSONIC ERROR CORRECTION

	Transmon Memory $T_{g,e}$	Cavity Memory $T_{0,1}$	Bosonic QEC lifetime $T_{0_L,1_L}$	Gain $G = \frac{T_{0_L,1_L}}{max(T_{g,e}, T_{0,1})}$
Cat codes [1]	$17 \pm 1\ \mu s$	$287 \pm 4\ \mu s$	$318 \pm 5\ \mu s$	**1.10**
Binomial codes [2]	$91 \pm 1\ \mu s$	$694 \pm 10\ \mu s$	$805 \pm 18\ \mu s$	**1.16**
GKP codes [3]	$250 \pm 1\ \mu s$	$800 \pm 10\ \mu s$	$1820 \pm 30\ \mu s$	**2.27**

References

Slides

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Slide 8-9: Cat code

Slide 10-13: Physical implementation of a Cat code

Slide 14-16: Binomial code

Slide 17-18: GKP code

Slide 17-20: Summary and progress of Bosonic QEC

References

[1] Michael A. Nielsen, Isaac L. Chuang. *Quantum Computation and Quantum Information, CH10* (Cambridge University Press, 2010)

[2] Cai, Weizhou, et al. "Bosonic quantum error correction codes in superconducting quantum circuits." *Fundamental Research* 1.1 (2021): 50-67.

[3] Ni, Zhongchu, et al. "Beating the break-even point with a discrete-variable-encoded logical qubit." *Nature* 616.7955 (2023): 56-60.

[4] Sun, Luyan, et al. "Tracking photon jumps with repeated quantum non-demolition parity measurements." *Nature* 511.7510 (2014): 444-448.

[5] Ofek, Nissim, et al. "Extending the lifetime of a quantum bit with error correction in superconducting circuits." *Nature* 536.7617 (2016): 441-445.

[6] Noh, Kyungjoo, and Christopher Chamberland. "Fault-tolerant bosonic quantum error correction with the surface–Gottesman-Kitaev-Preskill code." *Physical Review A* 101.1 (2020): 012316.

[7] Albert, Victor V., et al. "Performance and structure of single-mode bosonic codes." *Physical Review A* 97.3 (2018): 032346.

Thanks for listening to our presentation!